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The Impact of Pre-divorce and Post-divorce Counselling in Monogamous Marriages: A Mathematical Modelling Perspective

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Abstract: This study presents a deterministic compartmental model to investigate the impact of pre-divorce and post-divorce counselling on the dynamics of divorce within monogamous marriages. The model incorporates counselling as both a preventive and restorative mechanism and derives the threshold quantity R_0, which determines whether divorce persists or dies out in the population. Analytical results show that the divorce-free equilibrium is locally and globally asymptotically stable when $R_0 \leq 1$, whereas the divorce-endemic equilibrium becomes stable when $R_0 > 1$. The model further exhibits backward bifurcation, implying that reducing R_0 below one is not sufficient to eliminate divorce unless counselling interventions are intensified. Sensitivity analysis reveals that the divorce transmission rate (β) and pre-divorce counselling rate (ε) are the most influential parameters affecting R_0. Numerical simulations confirm the analytical results and demonstrate that strengthening counselling efforts reduces the prevalence of divorce substantially. The study highlights key policy implications and provides recommendations toward reducing marital instability through integrated counselling programmes.

Keywords: Pre-divorce counselling; Post-divorce counselling; Monogamous marriage; Stability; Sensitivity.

AMS Math Codes: 92D30; 37N25; 34D20; 92B05; 92D25.

1 Introduction

Mathematical models are helpful in understanding the behaviour of an infection whenever it enters a community [6–15]. Over the years, several models have been developed to investigate the dynamics of marriage and divorce in society. Specifically, Gweryina et al. [16] constructed a mathematical model to study the dynamics of divorce with anti-divorce therapy. They reported that the presence of marriage seminars promotes prolonged stable marriages and helps repair breaking points (separation and divorce). Gambrah and Adzadu developed a mathematical model to investigate the divorce epidemic in Ghana and concluded that educating couples experiencing separation is the most effective way to combat the spread of divorce in society. Duato and Jodar [17] formulated a mathematical model to examine the spread of divorce in Spain. Their study focused on a discrete linear–quadratic difference system model, with data collected from the Spanish Statistics Institute. They concluded that divorce is associated with economic and social contagion, as well as differences in happiness between spouses. Gambrah et al. [18] analysed the population dynamics of divorce and concluded that counselling sessions are useful in improving individuals' well-being and, consequently, the community. Hugo and Lusekelo [19] developed a mathematical model to investigate the dynamics of marriage and divorce scenarios, noting that counselling efforts reduce hardship and complexity among both married and divorced individuals. Syamsir et al. [20] revealed that reducing the rate of contact between married and divorced individuals and increasing the transfer rate from the split-bed class to the married class can minimise the spread of divorce. Abdul-Rahman and Alamri [21] indicated that differences in character, the amount of time spent on housework, and the level of attention wives give to their husbands contribute to the spread of divorce in society.

The growing relevance of counselling in managing marital conflict has highlighted the need for rigorous analytical tools to examine its impact on divorce dynamics. However, the combined influence of pre-divorce and post-divorce counselling on the long-term prevalence of divorce has not yet been adequately quantified in the mathematical modelling literature. Existing studies either ignore post-divorce counselling entirely, fail to incorporate counselling explicitly within the transition mechanisms of the model, or do not investigate how counselling interventions influence the basic reproduction number $\mathcal{R}_0$ and the possible existence of backward bifurcation. Moreover, to the best of our knowledge, no existing model integrates both pre-divorce and post-divorce counselling within a unified monogamous marriage framework. This gap underscores the need for a more comprehensive modelling approach that captures the full counselling process and its potential effects on marital stability.

In addressing this gap, the present study introduces a new compartmental model that incorporates explicit transitions for both pre-divorce and post-divorce counselling. The model enables the analytical derivation of the basic reproduction number $\mathcal{R}_0$ and offers a detailed examination of its behavioural and sociological implications. An important contribution of this work is the demonstration of backward bifurcation a phenomenon not previously reported in divorce-related models which has significant implications for the design of effective counselling interventions. The study further provides both local and global stability analyses, supported by sensitivity analysis using Partial Rank Correlation Coefficients (PRCCs) to identify the most influential parameters. Numerical simulations are also presented to illustrate and validate the theoretical findings, offering deeper insight into how counselling can mitigate the spread of divorce within a population.

The model formulated in this paper extends the general structure of divorce-dynamics models proposed in earlier works such as [18, 22]. However, unlike these models, we introduce two counselling-based transition mechanisms: (i) pre-divorce counselling that promotes reconciliation among temporarily separated couples, and (ii) post-divorce counselling that facilitates psychological recovery and reintegration. These additions render the present model structurally original, while still being inspired by existing compartmental modelling frameworks.

The rest of the paper is organised as follows: The assumptions and description of the model under consideration are presented in Section 2. In Section 3, the qualitative analysis and properties of the model are discussed. Sensitivity analysis is conducted in Section 5. In Section 6, the numerical simulations of the model are presented. Section 7 contains the findings of the study, and finally, the conclusions and recommendations are provided in Section 9.

2 The Model Formulation

The development of the model follows the compartmental modelling approach used in marriage and divorce dynamics (e.g., [18, 22]), but with significant modifications. Specifically, we incorporate two counselling mechanisms that alter

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the evolution of the marital states: pre-divorce counselling, which enables reconciliation among temporarily separated couples, and post-divorce counselling, which facilitates recovery among divorcees. These features distinguish the present model from existing ones and allow for a more realistic assessment of the influence of counselling on divorce prevalence.

The total population is divided into six (6) compartments: the susceptible compartment (S) represents single individuals who are ready to marry; the married compartment (M) denotes married individuals; the temporarily separated compartment (T_S) represents individuals who are separated but not divorced; the divorced compartment (D) denotes individuals who are divorced; the post-divorce counselled compartment (P_{dc}) represents individuals who have received post-divorce counselling; and the recovered compartment (R) denotes individuals who have recovered from divorce as a result of counselling.

To mathematically formulate the model, it is assumed that the rate at which people are recruited into the susceptible compartment is represented by ω. Susceptible individuals marry at the rate θ. Married individuals move into the temporarily separated compartment as a result of interactions between divorcees and married individuals at the rate βMD. Temporarily separated individuals rejoin the married compartment at the rate ε due to pre-divorce counselling. Temporarily separated individuals become divorced at the rate γ. Divorced individuals rejoin the susceptible class at the rate κ. Divorcees undergo post-divorce counselling at the rate τ. Those who receive post-divorce counselling recover at the rate d, whereas those who do not receive counselling recover at the rate r. Recovered individuals re-enter the susceptible compartment at the rate f. The parameters μ and χ represent the natural death rate and divorce-induced death rate, respectively. It is assumed that race, sex, and social status do not influence the probability of divorce and that individuals are homogeneously mixed.

The compartmental transitions in the model reflect key behavioural and sociological influences on marital stability. Interactions between married individuals and divorcees may increase marital instability through peer pressure, behavioural contagion, or social comparison effects, as described by Christakis and Fowler [23]. Pre-divorce counselling improves communication and reduces conflict, thereby preventing escalation. Post-divorce counselling facilitates psychological healing and social reintegration. These real-world mechanisms justify the transition terms incorporated into the proposed model.

The general dynamics of marriage–divorce interactions are depicted in Figure 1.

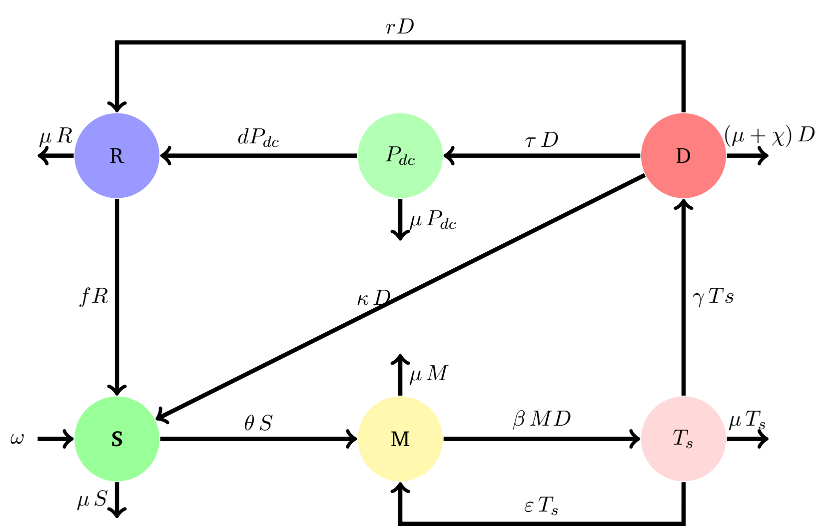

Figure 1: Schematic diagram of the dynamics of divorce with pre-divorce and post-divorce counselling

Table 1: Description of the Model Parameters

Parameter	Description
θ	The rate at which single individuals join the married population.
ω	The rate at which people are recruited into the susceptible class.
μ	The rate at which people die naturally in each compartment.
d	The rate at which divorcees recover due to counselling.
r	The rate at which divorcees recover without counselling.
τ	The rate at which divorced individuals go for counselling.
χ	The rate at which people die as a result of divorce.
β	The rate of contact between the married individuals and the divorcees.
κ	The rate at which divorce individuals rejoin the susceptible compartment.
ε	The rate at which people with marriage issues reconcile due to pre-divorce counselling.
γ	The rate at which temporarily-seperated individuals proceed to divorce their marriages.
f	The rate at which recovered individuals rejoin the susceptible compartment.

Table 2: The parameter Values of the Model

Parameter	Value	Source
θ	0.0400	[22]
ω	0.4000	[24]
μ	0.0150	[24]
τ	0.0120	Assumed
χ	0.0420	[17]
β	0.0300	[22]
κ	0.0303	[17], [18]
ε	0.0900	[22]
γ	0.0100	[17]
f	0.0010	Assumed
d	0.0200	Assumed
r	0.0220	Assumed

The descriptions of the model parameters are presented in Table 1

2.1 The Model Equations

The following ordinary differential equations describe the time evolution of population states represented in Figure (1).

$$\left.\begin{aligned}
\frac{dS}{dt} &= \omega + \kappa D + fR - (\theta + \mu)S, \\
\frac{dM}{dt} &= \theta S + \varepsilon T_s - (\mu + \beta D)M, \\
\frac{dT_s}{dt} &= \beta MD - (\mu + \varepsilon + \gamma)T_s, \\
\frac{dD}{dt} &= \gamma T_s - (\mu + \chi + \kappa + \tau + r)D, \\
\frac{dP_{dc}}{dt} &= \tau D - (\mu + d)P_{dc}, \\
\frac{dR}{dt} &= rD + dP_{dc} - (\mu + f)R
\end{aligned}\right\} \tag{2.1}$$

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3 Qualitative Properties

Some basic analytical properties of the model are presented in this section. The following result concerns the epidemiological feasibility of the model.

Theorem 3.1. *If the initial conditions of the model equations (2.1) are non-negatives, so are future solutions.*

Proof. From the first equation of (2.1), we have

$$\frac{\mathrm{d}S}{\mathrm{d}t} = \omega + \kappa D + fR - (\mu + \theta)S$$

This implies that

$$\frac{\mathrm{d}S}{\mathrm{d}t} \geq -(\mu + \theta)S \tag{3.1}$$

Integrating equation (3.1) gives

$$S(t) \geq S(0)e^{-(\mu+\theta)t}$$

Which implies

$$S(t) \geq 0$$

Similarly, it can be shown that $M(t) \geq 0$, $T_S(t) \geq 0$, $D(t) \geq 0$, $P_{dc}(t) \geq 0$, $R(t) \geq 0$ for all t > 0. From the analysis above, it is obvious that if the initial conditions are nonnegatives, then the future solutions are also nonnegatives. The proof is completed. □

The set containing the reasonable solutions of the model is known as the feasible region. The model predictions are shown to be bounded in the following theorem.

Theorem 3.2. *For all t ≥ 0, all solutions of the model equations (2.1) are uniformly bounded and contained in the feasible region defined by Ω = {(S(t), M(t), $T_S(t)$, D(t), $P_{dc}(t)$, R(t)) $\in \mathbb{R}^6_+ \mid 0 \leq N(t) \leq \frac{\omega}{\mu}$ }.*

Proof. Let (Ω) = {(S(t), M(t), $T_S(t)$, D(t), $P_{dc}(t), R(t)$) } $\in \mathbb{R}^6_+$ be the solutions of the model equations (2.1) with nonnegative conditions and let N = S + M + T_S+ D+ P_{dc} + R be the total population . Then

$$\frac{\mathrm{d}N}{\mathrm{d}t} = \frac{\mathrm{d}S}{\mathrm{d}t} + \frac{\mathrm{d}M}{\mathrm{d}t} + \frac{\mathrm{d}T_S}{\mathrm{d}t} + \frac{\mathrm{d}D}{\mathrm{d}t} + \frac{\mathrm{d}P_{dc}}{\mathrm{d}t} + \frac{\mathrm{d}R}{\mathrm{d}t} = \omega - \mu N - \chi D$$

Therefore

$$N(t) \leq \frac{\omega}{\mu} + \left(N_0 - \frac{\omega}{\mu}\right)e^{-\mu t}$$

and hence

$$\lim_{t\to\infty} N(t) \leq \frac{\omega}{\mu}$$

Thus, the solutions of the model are uniformly bounded and contained in the feasible set Ω = {(S(t), M(t), $T_S(t)$, D(t), $P_{dc}(t), R(t)) \in \mathbb{R}^6_+ \mid 0 \leq N(t) \leq \frac{\omega}{\mu}$ }. □

3.1 Equilibrium Points of the Model

The model has two equilibria, the divorce-free equilibrium $\mathcal{E}_0$ and a divorce equilibrium $\mathcal{E}^*$. The divorce-free equilibrium is given by $\mathcal{E}_0 = \left(\frac{\omega}{\mu+\theta},\ \frac{\theta\omega}{\mu(\mu+\theta)},\ 0,\ 0,\ 0,\ 0\right)$.

3.2 The Basic Reproduction Number $(\mathcal{R}_0)$

In marriage divorce, the basic reproduction number is the measure of the average number of married people influenced to divorce by a divorcee who enters a completely divorce-free community. The next generation matrix approach is applied to determine the basic reproduction number as described in [22].

Considering

$$\frac{\mathrm{d}T_S}{\mathrm{d}t} = \beta MD - (\mu + \varepsilon + \gamma)T_S$$
$$\frac{\mathrm{d}D}{\mathrm{d}t} = \gamma T_S - (\mu + \chi + \kappa + \tau + r)D$$

The transition and transmission matrices are respectively given by

$$\mathcal{F} = \begin{bmatrix} \beta MD \\ 0 \end{bmatrix} \text{ and } \mathcal{V} = \begin{bmatrix} (\mu + \varepsilon + \gamma)T_S \\ -\gamma T_S + (\mu + \chi + \kappa + \tau + r)D \end{bmatrix}$$

The Jacobian matrices at divorce-free equilibrium point are given by

$$F = \begin{bmatrix} 0 & \frac{\beta\theta\omega}{\mu(\theta+\mu)} \\ 0 & 0 \end{bmatrix} \text{ and } V = \begin{bmatrix} (\mu + \varepsilon + \gamma) & 0 \\ -\gamma & (\mu + \chi + \kappa + \tau + r) \end{bmatrix}$$

Therefore, the basic reproduction number is expressed as $\mathcal{R}_0 = \rho(FV^{-1}) = \frac{\beta\theta\omega\gamma}{\mu(\mu+\theta)(\mu+\varepsilon+\gamma)(\mu+\chi+\kappa+\tau+r)}$.

3.3 Divorce-endemic Equilibrium Point

The model can be shown to have a divorce-endemic equilibrium $(\mathcal{E}^*)$ given by:

$$\left.\begin{aligned} S^* &= \frac{\omega}{(\theta+\mu)} + \frac{1}{(\theta+\mu)}\left(\kappa + f\left(\frac{r(\mu+d)+\tau\, d}{(\mu+d)(\mu+f)}\right)\right)D^*; \\ M^* &= \frac{\theta\omega}{\mu(\theta+\mu)\mathcal{R}_0}; \\ T_s^* &= \frac{\beta\theta\omega}{\mu(\theta+\mu)(\mu+\varepsilon+\gamma)\mathcal{R}_0}D^*; \\ D^* &= \frac{\omega}{\Psi_1}\left(\frac{1}{\mathcal{R}_0} - 1\right); \\ P_{dc}^* &= \frac{\tau}{(\mu+d)}D^*; \\ R^* &= \left(\frac{r(\mu+d)+\tau\, d}{(\mu+d)(\mu+f)}\right)D^*. \end{aligned}\right\}$$

where,

$$\Psi_1 = \left[\kappa + f\left(\frac{r\,(\mu+d)+\tau\, d}{(\mu+d)\,(\mu+f)}\right) - \frac{\beta\omega\,(\mu+\gamma)}{\mu\,(\mu+\varepsilon+\gamma)\,\mathcal{R}_0}\right]$$

Now, for the divorce-persistent equilibrium to exist the following conditions must be met

1. Whenever $\mathcal{R}_0 > 1$ then Ψ_1 must be negative
2. Whenever $\mathcal{R}_0 < 1$ then Ψ_1 must be positive

The existence conditions for the divorce-endemic equilibrium reveal a crucial balance between marital reintegration flows and divorce generation pressure. When $\mathcal{R}_0 > 1$, the negative value of Ψ_1 indicates that divorce transmission overwhelms recovery mechanisms, leading to persistent marital instability. Conversely, when $\mathcal{R}_0 < 1$, a positive Ψ_1

signifies that counselling and reintegration programs successfully contain divorce spread. This balance underscores the importance of not only reducing divorce transmission but also strengthening post-divorce recovery pathways to ensure Ψ_1 remains positive, thereby preventing backward bifurcation where divorce persists despite $\mathcal{R}_0 < 1$.

3.4 Local Stability of Equilibria

The results on stability of the equilibria of the model are presented in Theorems 3.3, 3.4, and 3.5.

Theorem 3.3. *The divorce-free equilibrium point ($\mathcal{E}_0$) is locally asymptotically stable if $\mathcal{R}_0 \leq 1$ and unstable if $\mathcal{R}_0 > 1$.*

Proof. The Jacobian matrix of the model at ($\mathcal{E}_0$) is given by

$$J(\mathcal{E}_0) = \begin{bmatrix} -(\mu+\theta) & 0 & 0 & \kappa & 0 & f \\ \theta & -\mu & \varepsilon & -\frac{\beta\theta\omega}{\mu(\mu+\theta)} & 0 & 0 \\ 0 & 0 & -(\mu+\varepsilon+\gamma) & \frac{\beta\theta\omega}{\mu(\mu+\theta)} & 0 & 0 \\ 0 & 0 & \gamma & -(\mu+\chi+\kappa+\tau+r) & 0 & 0 \\ 0 & 0 & 0 & \tau & -(\mu+d) & 0 \\ 0 & 0 & 0 & r & d & -(\mu+f) \end{bmatrix}$$

Clearly, $\lambda_1 = -\mu,\ \lambda_2 = -(\mu+\theta),\ \lambda_3 = -(\mu+f),\ \ \lambda_4 = -(\mu+d)$ are eigenvalues of $J(\mathcal{E}_0)$, which are all negatives whenever $\mathcal{R}_0 < 1$ and the remaining eigenvalues are those of the following submatrix;

$$J^1(\mathcal{E}_0) = \begin{bmatrix} -(\mu+\varepsilon+\gamma) & \frac{\beta\theta\omega}{\mu(\mu+\theta)} \\ \gamma & -(\mu+\chi+\kappa+\tau+r) \end{bmatrix}$$

whose characteristic polynomial is given by

$$\lambda^2 + (2\mu+\varepsilon+\gamma+\kappa+\chi+r)\,\lambda + (\mu+\varepsilon+\gamma)\,(\mu+\kappa+\chi+r+\tau)\,(1-\mathcal{R}_0) = 0 \tag{3.2}$$

From equation (3.2), all the coefficients are positive whenever $\mathcal{R}_0 \leq 1$. It can be shown by Routh–Hurwitz criterion that, all the zeros of (3.2) have negative real parts. Therefore, the divorce-free equilibrium point ($\mathcal{E}_0$) is locally asymptotically stable if $\mathcal{R}_0 \leq 1$ and unstable if $\mathcal{R}_0 > 1$. □

Theorem 3.4. *The divorce-free equilibrium point ($\mathcal{E}_0$) is globally asymptotically stable whenever $\mathcal{R}_0 \leq 1$ and unstable if $\mathcal{R}_0 > 1$.*

Proof. Considering the Lyapunov function, $V = a_1 T + a_2 D$
It implies that,

$$\frac{dV}{dt} = a_1\frac{dT_S}{dt} + a_2\frac{dD}{dt} \tag{3.3}$$

Substituting $\frac{dT_S}{dt}$ and $\frac{dD}{dt}$ into equation (3.3) gives

$$\frac{dV}{dt} = a_1\left[\beta MD - (\mu+\varepsilon+\gamma)T_S\right] + a_2\left[\gamma T_S - (\mu+\chi+\kappa+\tau+r)D\right]$$

By expansion,

$$\frac{dV}{dt} = a_1\beta MD - a_2(\mu+\chi+\kappa+\tau+r)D + a_1\gamma T_S - (\mu+\varepsilon+\gamma)T_S$$

Taking $a_1 = \frac{\gamma}{(\mu+\chi+\kappa+\tau+r)}.a_2$

$$\frac{dV}{dt} = \frac{\gamma}{(\mu+\chi+\kappa+\tau+r)}.a_2\beta MD - a_2(\mu+\chi+\kappa+\tau+r)D \leq \frac{\beta\theta\omega\gamma}{\mu(\mu+\theta)(\mu_\varepsilon+\gamma)} - (\mu+\chi+\kappa+\tau+r)a_2$$

Putting $a_2 = 1$ gives

$$\frac{dV}{dt} = (\mu+\chi+\kappa+\tau+r)(\mathcal{R}_0 - 1)D$$

It is clear that $\frac{dV}{dt} \leq 0$ whenever $\mathcal{R}_0 \leq 1$ and $\frac{dV}{dt} = 0$ if and only if D = 0. Therefore, by Lasalle's invariance principle, the divorce-free equilibrium is globally asymptotically stable if $\mathcal{R}_0 \leq 1$ and unstable whenever $\mathcal{R}_0 > 1$. This completes the proof □

Theorem 3.5. *The divorce-endemic equilibrium point ($\mathcal{E}^*$) is locally asymptotically stable if $\mathcal{R}_0 > 1$ and unstable if $\mathcal{R}_0 \leq 1$.*

Proof. At the divorce-endemic equilibrium, the Jacobian matrix becomes

$$\mathcal{J} = \begin{bmatrix} -(\theta+\mu) & 0 & 0 & \kappa & 0 & f \\ \theta & -(\mu+\beta D^*) & \varepsilon & -\beta M^* & 0 & \\ 0 & \beta D^* & -(\mu+\varepsilon+\gamma) & \beta M^* & 0 & 0 \\ 0 & 0 & \gamma & -(\mu+\chi+\kappa+\tau+r) & 0 & 0 \\ 0 & 0 & 0 & \tau & (\mu+d) & 0 \\ 0 & 0 & 0 & r & d & -(\mu+f) \end{bmatrix}$$

We linearize the system around $\mathcal{E}^*$ and examine the corresponding Jacobian matrix $\mathcal{J}(\mathcal{E}^*)$. From the model formulation, $\mathcal{J}(\mathcal{E}^*)$ can be partitioned as

$$\mathcal{J}(\mathcal{E}^*) = \begin{pmatrix} A & 0 \\ 0 & B \end{pmatrix},$$

where A is the 4×4 submatrix associated with the state variables (S, M, TS, D), and B is the 2×2 submatrix corresponding to the post-divorce counselling (P_{dc}) and recovered (R) compartments. Explicitly, one obtains

$$A = \begin{pmatrix} -(\theta+\mu) & 0 & 0 & \kappa \\ \theta & -(\mu+\beta D^*) & \varepsilon & -\beta M^* \\ 0 & \beta D^* & -(\mu+\varepsilon+\gamma) & \beta M^* \\ 0 & 0 & \gamma & -(\mu+\chi+\kappa+\tau+r) \end{pmatrix}, \qquad B = \begin{pmatrix} -(\mu+d) & 0 \\ d & -(\mu+f) \end{pmatrix}.$$

The submatrix B has two negative eigenvalues $\lambda_5 = -(\mu+d)$ and $\lambda_6 = -(\mu+f)$. Consequently, the characteristic polynomial of $\mathcal{J}(\mathcal{E}^*)$ factorizes as

$$\det(\lambda I - \mathcal{J}(\mathcal{E}^*)) = (\lambda+\mu+d)(\lambda+\mu+f)\,Q(\lambda),$$

where $Q(\lambda) = \det(\lambda I_4 - A)$ is a quartic polynomial of the form

$$Q(\lambda) = \lambda^4 + a_1\lambda^3 + a_2\lambda^2 + a_3\lambda + a_4. \tag{3.4}$$

Thus, local stability of $\mathcal{E}^*$ is determined by the signs of the real parts of the roots of $Q(\lambda)$. Evaluating the determinant

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$\det(\lambda I_4 - A)$ yields the following coefficients in terms of the model parameters and the endemic state values M^* and D^*:

$$
\begin{aligned}
a_1 &= \beta D^* + \chi + \varepsilon + \gamma + \kappa + 4\mu + r + \tau + \theta, \\
a_2 &= \beta D^*(\chi + \gamma + \kappa + 3\mu + r + \tau + \theta) - \beta M^* \gamma \\
&\quad + \chi\varepsilon + \chi\gamma + 3\chi\mu + \chi\theta + \varepsilon\kappa + 3\varepsilon\mu + \varepsilon(r + \tau + \theta) \\
&\quad + \gamma\kappa + 3\gamma\mu + \gamma(r + \tau + \theta) + 3\kappa\mu + \kappa\theta + 6\mu^2 + 3\mu(r + \tau + \theta) + r\theta + \tau\theta, \\
a_3 &= \beta D^*\big(\chi\gamma + 2\chi\mu + \chi\theta + \gamma\mu + \gamma\theta + 2\mu^2 + \mu r + \mu\tau + \mu\theta\big) \\
&\quad - \beta M^* \gamma(\mu + \theta) \\
&\quad + \chi\varepsilon\mu + \chi\gamma\mu + \chi\mu^2 + \chi\mu\theta \\
&\quad + \varepsilon\kappa\mu + \varepsilon\mu^2 + \varepsilon\mu(r + \tau + \theta) \\
&\quad + \gamma\kappa\mu + \gamma\mu^2 + \gamma\mu(r + \tau + \theta) \\
&\quad + \kappa\mu^2 + \kappa\mu\theta \\
&\quad + 3\mu^3 + 3\mu^2(r + \tau + \theta) + 2\mu(r\theta + \tau\theta), \\
a_4 &= \Big\{ - \beta\theta d f \gamma\tau - \beta(\mu + \varepsilon + \gamma)(\mu + \theta)(\mu + d)(\mu + f)\,\varepsilon \\
&\quad - \beta\theta f \gamma(\mu + d)\, r - \beta\theta\gamma(\mu + d)(\mu + f)\,\kappa \\
&\quad + \beta(\mu + \varepsilon + \gamma)(\mu + \chi + \kappa + \tau + r)(\mu + \theta)(\mu + d)(\mu + f)\Big\} D^* \\
&\quad - M^*\,\beta\gamma(\mu + \varepsilon + \gamma)(\mu + d)(\mu + f)\,\mu \\
&\quad + (\mu + \varepsilon + \gamma)(\mu + \chi + \kappa + \tau + r)(\mu + \theta)(\mu + d)(\mu + f)\,\mu.
\end{aligned}
$$

For the quartic equation 3.4 the Routh–Hurwitz conditions for all roots to have negative real parts are:

(i) $a_1, a_2, a_3, a_4 > 0$,

(ii) $a_1 a_2 > a_3$,

(iii) $a_1 a_2 a_3 > a_1^2 a_4 + a_3^2$.

Substituting the parameter expressions for a_i confirms that these inequalities are satisfied whenever $R_0 > 1$. Each term in the coefficients represents a sum or product of positive rates and positive equilibrium values, ensuring positivity of the Hurwitz determinants. Because the two eigenvalues from B are real and negative, and the quartic $Q(\lambda)$ satisfies the Routh–Hurwitz conditions, all eigenvalues of $\mathcal{J}(\mathcal{E}^*)$ have negative real parts when $R_0 > 1$. Hence, the divorce-endemic equilibrium $\mathcal{E}^*$ is locally asymptotically stable whenever $R_0 > 1$. □

4 Bifurcation

The parameter β, representing the divorce transmission rate, is selected as the bifurcation parameter due to its central role as the primary driver of marital instability. It directly governs the social contagion of divorce and exhibits a one-to-one relationship with the basic reproduction number $\mathcal{R}_0$. Analyzing bifurcation with respect to β provides critical insight into the threshold behaviour of the system and identifies the level of social influence that must be managed to prevent a self-sustaining divorce epidemic.

Taking β as the bifurcation parameter, the Jacobian of the system evaluated at the divorce-free equilibrium will have

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a simple eigenvalue, with the associated right and left eigenvectors given by

$$\mathbf{w} = (w_1,\ w_2,\ w_3,\ w_4,\ w_5,\ w_6) \text{ and } \mathbf{v} = \left(0,\ 0,\ \frac{\gamma\, v_4}{\varepsilon+\gamma+\mu},\ v_4,\ 0,\ 0\right);$$

where

$$\begin{aligned} w_1 &= \tfrac{1}{\theta+\mu}\left[\kappa + f\left(\tfrac{r(d+\mu)+d\,\tau}{(d+\mu)(f+\mu)}\right)\right] w_4, \\ w_2 &= \theta\, w_1 - \tfrac{w_4\,(\gamma+\mu)\,(\tau+\chi+\kappa+\mu+r)}{\gamma}, \\ w_3 &= \tfrac{w_4\,(\tau+\chi+\kappa+\mu+r)}{\gamma}, \\ w_5 &= \tfrac{\tau\, w_4}{d+\mu}, \\ w_6 &= \left(\tfrac{r(f+\mu)+d\,\tau}{(d+\mu)(f+\mu)}\right) w_4 \end{aligned}$$

$$\mathbf{a} = \frac{2\beta^*\gamma}{\mu\,(\varepsilon+\gamma+\mu)}\left[\frac{\theta}{\theta+\mu}\left(\kappa + f\left(\frac{r\,(d+\mu)+d\,\tau}{(d+\mu)\,(f+\mu)}\right)\right) - \frac{(\gamma+\mu)\,(\tau+\chi+\kappa+\mu+r)}{\gamma}\right] v_4 w_4^2$$

$$\mathbf{b} = \frac{\theta\omega\,\gamma\, v_4\, w_4}{\mu\,(\mu+\theta)\,(\varepsilon+\gamma+\mu)}$$

The analysis shows that since $\mathbf{a} > 0$ and $b > 0$, the model undergoes a backward bifurcation at the critical threshold $\mathcal{R}_0 = 1$, in accordance with the bifurcation framework proposed by Castillo and Song [25]. This result implies that divorce may persist even when $\mathcal{R}_0 < 1$, meaning that merely reducing the basic reproduction number below unity is insufficient to eradicate divorce completely. Therefore, comprehensive and sustained counselling interventions, both pre-divorce and post-divorce, are essential to suppress divorce prevalence and guide the system toward a stable, divorce-free equilibrium.

5 Sensitivity Analysis

Sensitivity analysis generally informs us how specific changes in each parameter influence the spread of divorce in the community. To conduct the sensitivity analysis in this study, the Normalised Sensitivity Index Method is employed, as presented in [26,27]. According to [28], the normalised sensitivity index of a variable $\mathcal{R}_0$, which depends differentiably on a parameter q, is mathematically defined as:

$$\Upsilon_q^{\mathcal{R}_0} = \frac{\partial \mathcal{R}_0}{\partial q} \times \frac{q}{\mathcal{R}_0}.$$

Considering the parameter, ω, χ, and $\mathcal{R}_0 = \frac{\beta\theta\omega g}{\mu(\mu+\theta)(\mu+\varepsilon+\gamma)(\mu+\chi+\kappa+\tau+r)}$, the sensitivity index of the basic reproduction number, $\mathcal{R}_0$, with respect to the parameters:

$$\Upsilon_\omega^{\mathcal{R}_0} = \frac{\partial \mathcal{R}_0}{\partial \omega} \times \frac{\omega}{\mathcal{R}_0} = 1$$

$$\Upsilon_\beta^{\mathcal{R}_0} = \frac{\partial \mathcal{R}_0}{\partial \beta} \times \frac{\beta}{\mathcal{R}_0} = 1$$

$$\Upsilon_\chi^{\mathcal{R}_0} = \frac{\partial \mathcal{R}_0}{\partial \chi} \times \frac{\chi}{\mathcal{R}_0} = \frac{-\chi}{(\mu+\chi+\kappa+\tau+r)} = -0.3462$$

$$\Upsilon_\kappa^{\mathcal{R}_0} = \frac{\partial \mathcal{R}_0}{\partial \kappa} \times \frac{\kappa}{\mathcal{R}_0} = \frac{-\kappa}{(\mu+\chi+\kappa+\tau+r)} = -0.2498$$

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$$\Upsilon_{\tau}^{\mathcal{R}_0} = \frac{\partial \mathcal{R}_0}{\partial \tau} \times \frac{\tau}{\mathcal{R}_0} = \frac{-\tau}{(\mu + \chi + \kappa + \tau + r)} = -0.0989$$

$$\Upsilon_{\varepsilon}^{\mathcal{R}_0} = \frac{\partial \mathcal{R}_0}{\partial \varepsilon} \times \frac{\varepsilon}{\mathcal{R}_0} = \frac{-\varepsilon}{(\mu + \varepsilon + \gamma)} = -0.7826$$

Similarly, the sensitivity indices of the basic reproduction number, $\mathcal{R}_0$, with respect to the rest of the parameters are computered and captured in Table 3.

Table 3: Sensitivity Index of $\mathcal{R}_0$ Parameters

Parameter	Sensitivity Index
β	+1.0000
ω	+1.0000
θ	+0.6340
γ	-0.0870
μ	-0.4210
τ	-0.0989
χ	-0.3462
κ	-0.2498
ε	-0.7826
r	-0.1814

From Table 3, the parameters that have positive sensitivity indices (β, θ, and ω) indicate that they have a great influence on the spread of divorce in the community whenever their values are increased. Also, those parameters with negative sensitivity indices (χ, ε, κ, τ, r, and μ) show that they have an effect of reducing the spread of divorce in the community whenever their values are increased. This implies that all the parameters with negative sensitivity indices should be increased whereas those with positve sensitivity indices should be decreased in order to minimise the spread of divorce in Ghana, Africa, and around the world.

6 Numerical Simulation

In this section, the numerical simulations of the model are presented and analysed to confirm the analytical results of the study. MATLAB ODE54 Solver is considered and the following are used as the initial conditions: S(0)=1000, M(0)=100, $T_S(0) = 50$, D(0)=20, $P_{dc}(0) = 10$, R(0)=5 and the Parameter values used to carry out the various simulations are captured in Table 2.

Figure 2 presents a series of contour diagrams illustrating the variation of the basic reproduction number, R_0, with respect to key model parameters. Each subplot visualizes how the interplay between parameters such as the divorce transmission rate (β), the marriage rate (θ), the pre-divorce counselling rate (ε), the post-divorce counselling rate (τ), and the divorce-induced mortality rate (χ) influences the magnitude of R_0.

From the contours, it is observed that R_0 increases monotonically with higher values of β and θ, indicating that greater marital interactions and increased marriage rates enhance the potential for divorce transmission within the population. Conversely, increases in ε, τ, and χ tend to suppress R_0, revealing that intensified pre-divorce counselling, active post-divorce counselling programmes, and divorce-related attrition effectively reduce the propagation of divorce. The contours provide a visual sensitivity map: parameters in the numerator $(\beta, \theta, \omega, \gamma)$ exert a positive influence on R_0, while those in the denominator $(\mu, \varepsilon, \gamma, \chi, \kappa, \tau, r)$ yield negative effects. The parameter spaces delineated in Figure 2 therefore demarcate the stability threshold between the divorce-free regime $(R_0 < 1)$ and the divorce-endemic regime

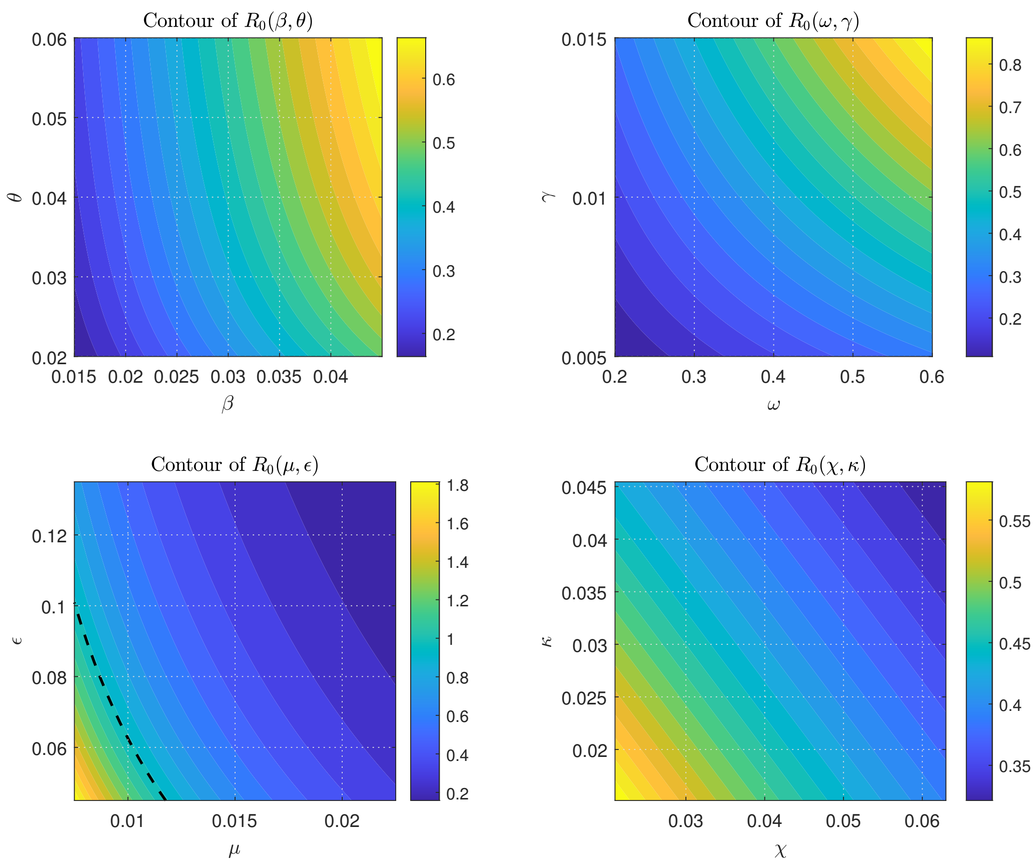

Figure 2: Contour Plots of the Basic Reproduction Number R_0

$(R_0 > 1)$. The upward-sloping isolines show that small simultaneous increments in both β and θ can rapidly push the system across the epidemic threshold, underscoring the need for strategic counselling interventions to dampen divorce contagion.

Figure 3 displays the Partial Rank Correlation Coefficients (PRCCs) of the model parameters with respect to R_0. PRCC quantifies the strength and direction of the monotonic relationship between each parameter and the model output after accounting for the influence of other parameters. A PRCC value close to $+1$ (or -1) signifies a strong positive (or negative) correlation with the basic reproduction number.

The results indicate that parameters β, θ, and ω exhibit high positive PRCC values, confirming that increments in these parameters significantly elevate R_0, thereby intensifying divorce transmission. Conversely, parameters ε, τ, κ, χ, r, and μ exhibit negative PRCC values, implying that enhancing counselling efficacy, reintegration, recovery, and divorce-related mortality can substantially lower R_0.

The ranking of the PRCC magnitudes reveals that ε (pre-divorce counselling rate) and β (divorce transmission rate) are the most influential parameters. This analytical insight aligns with the contour-based findings in Figure 2, emphasizing that early intervention through counselling and reduction in divorce exposure are critical levers for maintaining marital stability.

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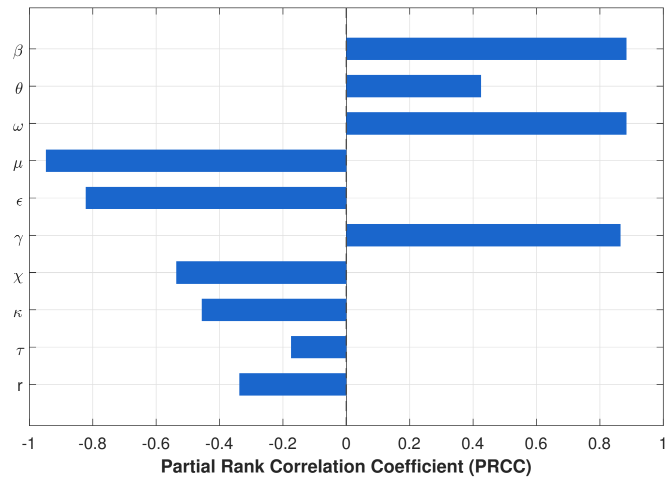
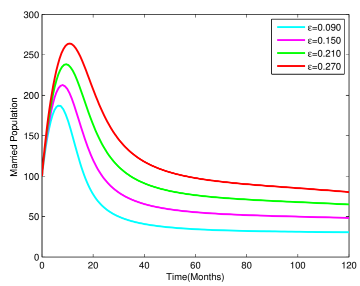
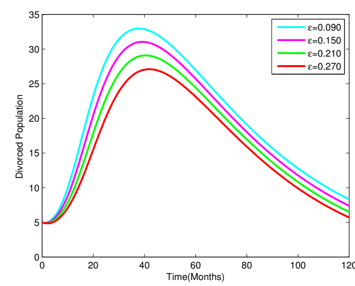


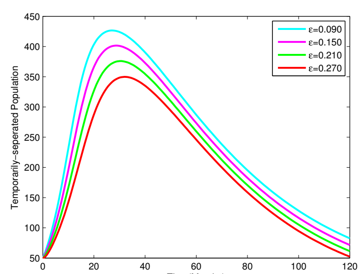
Figure 3: PRCC Sensitivity Analysis of Model Parameters on R_0



(a) Trajectories of model solution for varying rates of ε on married population



(b) Trajectories of model solution for varying rates of ε on divorced population



(c) Trajectories of model solution for varying rates of ε on the temporarily-separated population

Figure 4: The impact of pre-divorce Counselling on the married, divorced, and temporarily-separated populations

It is observed in Figure 4 that, whenever the rate of pre-divorce counselling (ε) is increased, the number of married individuals increased while the number of divorcees and the temporarily-separated individuals decreased and vice visa. This means that couples who are separated should always consider pre-divorce counselling in order to reconcile their marriage. This will help to minimise the spread of divorce in the society.

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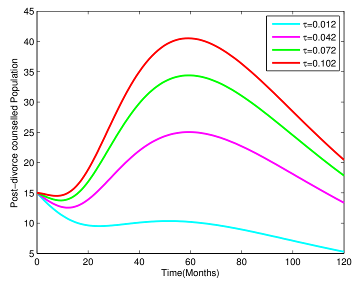
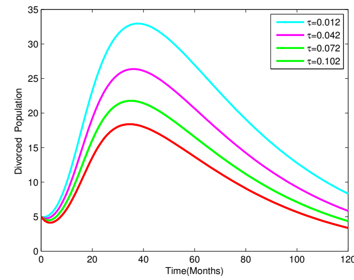
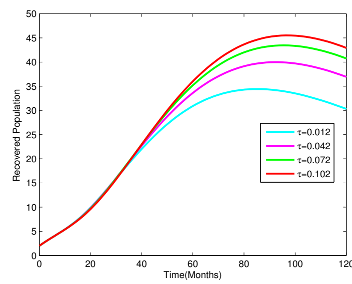


(a) Trajectories of model solution for varying rates of τ on the Post-divorce Counselled population



(b) Trajectories of model solution for varying rates of τ on the divorced population



(c) Trajectories of model solution for varying rates of τ on the recovered population

Figure 5: The Impact of post-divorce counselling on the Post-divorce counselled, the divorced, and the recovered populations

It is shown in Figure 5 that, whenever the rate of post-divorce counselling (τ) is increased, the number of recovered and post-divoce counselled individuals increased while the number of divorcees decreased. This implies that post-divorce counselling should be considered in order to assist divorcees to recover from the stress and trauma associated with divorce. This will help reduce the number of suicidal cases among divorcees in the society.

7 Discussion

The divorce-endemic model developed in this study provides a theoretical and numerical framework for understanding the transmission dynamics of divorce within a monogamous population. The analysis integrates counselling dynamics into the mathematical structure of the model and explores how counselling interventions, marital interactions, and recovery mechanisms influence the persistence or elimination of divorce. Figures 1–5 collectively provide a visual and quantitative synthesis of the model's structure, threshold behaviour, parameter sensitivity, and dynamic responses under various counselling strategies.

Figure 1 presents the schematic flow diagram of the divorce-endemic model, illustrating the transitions among the various marital states, namely the married, temporarily separated, divorced, post-divorce counselled, and recovered populations. The diagram provides a conceptual understanding of how individuals move through these compartments depending on social and counselling factors. The flowchart further reveals the pathways through which preventive and post-divorce counselling interact with the core divorce dynamics, reinforcing the idea that counselling serves as both a preventive and corrective mechanism for restoring marital stability.

Figure 2 depicts the contour plots of the basic reproduction number, R_0, as a function of selected parameter pairs.

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These plots highlight the nonlinear sensitivity of R_0 to changes in the divorce transmission rate (β), marriage rate (θ), and counselling-related parameters $(\varepsilon, \tau, \chi)$. The contours show that R_0 increases with higher values of β and θ, suggesting that frequent marital exposure and high union formation rates increase the likelihood of divorce spread. In contrast, higher values of ε (pre-divorce counselling rate), τ (post-divorce counselling rate), and χ (divorce-induced attrition rate) reduce R_0, indicating that effective counselling programs and reintegration mechanisms are essential for mitigating divorce propagation. The dashed contour line at $R_0 = 1$ separates the divorce-free region from the divorce-endemic region, illustrating the critical threshold for maintaining marital equilibrium within the population. Figures 4 and 5 show the time-series simulations illustrating the temporal impact of counselling interventions on the evolution of the married, divorced, and post-divorce populations.

The Partial Rank Correlation Coefficient (PRCC) results presented in Figure 3 provide crucial insights into the key drivers of divorce dynamics, revealing a clear dichotomy between risk-amplifying and protective parameters. The parameters β (divorce transmission rate), θ (marriage rate), and ω (recruitment rate) exhibit strong positive correlations with $\mathcal{R}_0$, indicating that social contagion effects and rapid marriage formation without adequate support structures significantly amplify divorce spread. Conversely, the pre-divorce counselling rate (ε) emerges as the dominant protective factor, demonstrating the highest magnitude negative correlation with $\mathcal{R}_0$ and underscoring the critical importance of early intervention. Post-divorce counselling (τ) and reintegration mechanisms (κ) also show significant negative correlations, though with lesser magnitude, highlighting the value of restorative interventions even after divorce occurs. Collectively, these findings advocate for a dual-strategy policy approach: implementing public health campaigns to reduce social contagion of divorce while prioritizing accessible pre-divorce counselling services as the most effective intervention, supplemented by post-divorce support programs to facilitate recovery and reintegration. This hierarchical intervention strategy, focusing resources on the most sensitive parameters, offers the optimal pathway to reduce marital instability within populations.

Figure 4 focuses on the influence of pre-divorce counselling on the married, divorced, and temporarily separated classes. The simulation demonstrates that as the rate of pre-divorce counselling (ε) increases, the divorced and temporarily separated populations decline significantly, while the married population rises. This behaviour underscores the preventive power of counselling in averting marital breakdowns before they escalate to permanent separation. By stabilising couples during early conflict stages, pre-divorce counselling effectively maintains a higher proportion of married individuals in the system. Figure 5 illustrates the impact of post-divorce counselling on the post-divorce counselled, divorced, and recovered populations. The simulation reveals that increasing the post-divorce counselling rate (τ) leads to a notable reduction in the number of permanently divorced individuals and a corresponding rise in the recovered and reintegrated classes. This trend demonstrates that even after marital dissolution, counselling interventions can play a restorative role by facilitating reconciliation, psychological healing, or social reintegration. The long-term effect is a reduction in the overall divorce burden and a gradual movement of the system toward marital recovery equilibrium.

8 Limitations and Future Research Directions

Although the model provides valuable insights, it is subject to several limitations:

- The assumption of homogeneous mixing ignores structured social interactions.
- Model parameters are assumed constant, although counselling effectiveness varies over time.
- The model does not incorporate age or gender structure.

- Socio-economic and cultural heterogeneity is not included.
- The model assumes deterministic behaviour, ignoring stochastic fluctuations.

Future work may incorporate age structure, time-varying counselling interventions, socio-economic heterogeneity, stochastic dynamics, or fractional-order derivatives to capture memory effects in marital interactions.

9 Conclusion and Policy Implications

This study developed and analysed a deterministic divorce-endemic model that incorporates both pre-divorce and post-divorce counselling as intervention strategies to control marital instability within monogamous societies. Through analytical derivations and numerical simulations, the model elucidates the critical factors that influence the basic reproduction number, R_0, which serves as a threshold indicator for the persistence or elimination of divorce in the population. The results demonstrate that the divorce-endemic equilibrium is locally asymptotically stable when $R_0 > 1$ and that the system transitions toward a divorce-free equilibrium when $R_0 < 1$. These theoretical findings are supported by the contour plots, sensitivity analyses, and time-series simulations, all of which reveal that counselling-related parameters exert the most significant influence on the reduction of R_0.

The sensitivity analysis identifies the pre-divorce counselling rate (ε) and the divorce transmission rate (β) as the most influential parameters driving the system dynamics. An increase in ε effectively reduces the value of R_0, highlighting the crucial role of counselling in preventing marital breakdowns. Similarly, enhancing post-divorce counselling (τ) reduces the long-term proportion of divorced individuals and facilitates their reintegration into the married or recovered populations. These findings collectively affirm that counselling interventions function not only as preventive mechanisms but also as restorative processes capable of reversing the social and psychological impacts of divorce.

From a policy perspective, the outcomes of this study underscore the necessity of institutionalising counselling programs as part of national family welfare systems. Governments, faith-based organisations, and community leaders should prioritise the establishment of accessible pre- and post-divorce counselling centres to mitigate rising divorce rates. Integrating counselling services into marital registration processes and community health programs would enhance early detection of marital conflicts and facilitate timely intervention. Furthermore, training and certification of professional counsellors should be expanded to ensure that counselling practices are evidence-based, culturally sensitive, and widely accessible.

The model also provides a framework for evaluating the socio-dynamic benefits of counselling within mathematical and policy contexts. By quantifying the threshold behaviour of divorce through R_0, policymakers can assess the minimum level of intervention required to achieve social stability. In this regard, the mathematical insights offer a novel decision-support tool for designing targeted strategies that balance preventive and corrective measures.

Declarations

Author's Contributions: Abagna Stephe: Conceptualization, Design, Investigation, Analysis, Numerical simulation, Writing-original draft. Philip N. A. Akuka: Conceptualization, Design, Investigation, Analysis, Numerical simulation, Writing-original draft. Baba Seidu: Supervision, Editing, Proofreading, Numerical simulation. Christopher S. Bornaa:Supervision, Editing, Proofreading, Numerical simulation. Eric Okyere: Writing- review & editing.

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