Journal of Mathematical Epidemiology, 1(2) (2026), 131-153.

Research Paper
Open Access
ISSN: 3108-4052

# Modeling the Evolution of Xenophobic Attitudes: A Fractional-Order Approach and Optimal Strategies

Gizachew Kefelew Hailu[1,*], Shewafera Wondimagegnhu Teklu[1], Dejen Ketema Mamo[1]

[1]Debre Berhan University, Debre Berhan, Ethiopoia

*Corresponding author e-mail: kgizachewy@gmail.com

Doi: https://doi.org/10.64891/jome.8

**Abstract:** Xenophobia persists as a significant societal challenge with profound impacts on community cohesion, yet conventional policy approaches often prove insufficient. This research presents a Caputo fractional-order model to examine xenophobia dynamics, capturing memory effects. The model classifies populations into five distinct groups: Protected (P), Susceptible (S), Exposed (E), Affected (A), and Recovered (R). We rigorously establish solution positivity and boundedness, identify both xenophobia-free and endemic equilibrium states, and conduct comprehensive stability analysis to characterize long-term behavioral patterns. The study derives a threshold reproduction number governing xenophobia spread dynamics and formulates an associated optimal control problem. This framework evaluates two complementary intervention strategies: socio-economic initiatives ($w_1(t)$) addressing poverty and inequality, and legal-judicial measures ($w_2(t)$) strengthening law enforcement. Numerical simulations reveal that coordinated implementation ($w_1 = w_2 = 0.7$) significantly reduces xenophobia prevalence, while lower fractional orders ($\kappa \to 0.5$) demonstrate enhanced recovery rates due to memory effects. These results strongly support integrated policy interventions combining socio-economic and legal approaches to effectively mitigate xenophobic transmission and foster inclusive communities.

**Keywords:** Xenophobia; Mathematical model; Fractional-order derivative; Optimal control; Socio-economic interventions.
**AMS Math Codes:** 49Q12; 37N25.

## 1 Introduction

Xenophobia, derived from the ancient Greek words *xenos* (meaning foreign) and *phobos* (meaning fear), refers to the fear of foreigners. It is commonly understood as a hostility towards foreigners and fear of people from other nations, often manifesting as hatred and prejudice against them [1]. Factors contributing to xenophobia include economic conditions, regional migration trends, fears of cultural threats, political instability, religious beliefs, and terrorism [2]. Its manifestations range from non-violent discrimination and segregation [3], xenophobic violence [4–6], intense dislike, hatred or fear of others [1], and isolation [7]. The implications of xenophobia extend beyond individual experiences; they affect social cohesion, economic stability, and national security. Therefore, understanding and controlling xenophobic attitudes is of paramount importance for fostering inclusivity and harmony in increasingly multicultural societies.

Traditional approaches to addressing xenophobia often focus on educational programs, community engagement, and policy reform. Vhumbunu and et al [8] found in their research that, although substantive policies, laws, action plans, and institutional frameworks have been developed and implemented at the national, provincial, and local levels to tackle xenophobia in South Africa, these measures lack the comprehensive and community-focused approach needed to address the structural and underlying causes of xenophobia in the country. This highlights the need for innovative and systematic methods to gain deeper insights into the underlying mechanisms that drive xenophobic attitudes and to develop more effective strategies for their control, increasing xenophobic orientations.

Recent studies have modeled real-world systems using either integer-order derivatives [3, 9–11] or fractional-order methods [12–14]. Fractional order modeling (FOM) has emerged as a powerful tool for analyzing complex dynamic systems across various fields, including engineering, biology, and economics [15]. Unlike traditional integer-order models, fractional order models can capture memory effects and non-local interactions, making them particularly suitable for representing the intricate processes involved in human behavior and social dynamics. By employing a fractional order approach, we can better understand the temporal evolution of xenophobic attitudes and identify critical points for intervention.

Moreover, optimal control strategies provide a framework for systematically designing interventions aimed at minimizing xenophobic attitudes over time. Optimal control theory, which originated in the field of mathematics and engineering, seeks to determine the best course of action in dynamic systems by optimizing a predefined objective function. By integrating optimal control with fractional order modeling, we can formulate targeted strategies that not only reduce xenophobia but also promote positive social interactions and inclusivity.

This study aims to explore the potential of combining fractional order modeling with optimal control strategies to gain insights into controlling xenophobic attitudes. The objectives of this research are threefold: first, to develop a comprehensive fractional order model that captures the dynamics of xenophobic attitudes within a given population; second, to apply optimal control techniques to design effective interventions for mitigating these attitudes; and third, to analyze the outcomes of various control strategies in order to identify the most effective approaches for promoting inclusivity.

By employing a rigorous mathematical approach to the study of xenophobia, this research aims to contribute to the growing body of literature on social dynamics and provide practical solutions for addressing one of the most pressing social issues of our time. The integration of fractional order modeling with optimal control strategies not only offers a novel perspective on understanding xenophobic attitudes but also lays the groundwork for evidence-based interventions that can foster greater societal cohesion and acceptance.

The remaining sections of the paper are presented as follows: In section 2, we introduced some fundamental ideas, definitions, and propositions of fractional-order derivatives that were employed in the study of the model. In section 3, we formulated both integer order and fractional order models, demonstrated the positivity and boundedness of the model's solution, identified equilibrium states, and performed stability analysis on the Caputo model. In section 4, we formulated the optimal control problem and conducted numerical simulation. Discussions of the results of the numerical solution, concluding remarks and potential next steps are presented in section 5.

Research Paper
Open Access
ISSN: 3108-4052

# 2 Preliminaries

The following concepts of fractional order calculus will serve as a foundation for constructing the fractional order model in this study.

**Definition 2.1.** *For a function $f \in \mathbb{C}^n$, the Caputo derivative with a fractional order $\kappa$ is defined by [16, 17]*

$$D_t^{\kappa} f(t) = \frac{1}{\Gamma(n-\kappa)} \int_0^t f^n(\zeta) \frac{(t-\zeta)^{n-\kappa}}{t-\zeta}\, d\zeta,\ n-1 < \kappa \le n \in \mathbb{N}$$

**Note**: $D_t^{\kappa} f(t)$ tends to $f(t)$ as $\kappa \to 1$

**Definition 2.2.** *For a function $f \in C^n$, the Caputo integral with a fractional order $\kappa > 0$ is defined by [16, 17]*

$$I_t^{\kappa} f(t) = \frac{1}{\Gamma(\kappa)} \int_0^t f(\zeta) \frac{(t-\zeta)^{\kappa}}{t-\zeta}\, d\zeta,\ 0 < \kappa < 1, t > 0$$

**Definition 2.3.** *The Mittag-Leffler function with two parameters is defined by [16, 17]*

$$E_{\alpha,\beta}(t) = \Sigma_{k=1}^{\infty} \frac{t^k}{\Gamma(\alpha k + \beta)} \text{ where } \alpha, \beta > 0$$

**Definition 2.4.** *For $\beta = 1$, the Mittag-Leffler function with one parameter is defined as [16]*

$$E_{\alpha}(t) = E_{\alpha,1}(t) = \Sigma_{k=1}^{\infty} \frac{t^k}{\Gamma(\alpha k + 1)}$$

**Definition 2.5.** *A fixed number $\theta^*$ is recognized as a steady state (or an equilibrium point) of the Caputo-fractional order model when*

$$D_t^{\kappa}\theta(t) = f\big(t, \theta(t)\big), \kappa \in [0,1] \text{ if and only if } f(t, \theta^*) = 0$$

**Proposition 2.6.** *The Laplace transform of the Caputo fractional order derivative with order $\kappa$, $n-1 < \kappa \le n$, $n \in N$ is given by $\mathcal{L}(D_t^{\kappa} f)(s) = s^{\kappa} F(s) - \Sigma_{i=0}^{n-1} s^{\kappa-i-1} f^i(0)$ where $F(s)$ is the Laplace transform of the function $f(t)$ [15].*

**Proposition 2.7.** *The Laplace transform of the Mittag Leffler function in two parameters is given by*

$$\mathcal{L}\big(t^{\beta-1} E_{\alpha,\beta}(\pm\gamma t^{\alpha})\big)(s) = \frac{s^{\alpha-\beta}}{s^{\alpha} \mp \gamma}$$

**Proposition 2.8** (Generalized Mean Value Theorem)**.** *Suppose $h(t) \in L[0, T_f]$ and $D_t^k h(t) \in L[0, T_f]$ for $k \in (0,1]$ then the theorem states that $h(t) = h(0) + \frac{1}{\Gamma(k)} D_t^k h(\zeta)^t$ where $\zeta \in [0,t]$, for each $t$ such that $0 < t \le T_f$.*

**Lemma 2.9.** *The two statements below are derived from Proposition 2.8 a) The function $h$ is non-decreasing for all $t \in [0, T_f]$, if ${}^C D_t^k h(t) \ge 0$. b) The function $h$ is non-increasing for all $t \in [0, T_f]$, if ${}^C D_t^k h(t) \le 0$.*

**Proposition 2.10.** *Suppose $g(t) \in L^{\infty}(\mathbb{R}) \cap \mathcal{F}(\mathbb{R})$ and $k \in \mathbb{R}$, $n-1 < k \le n, n \in N$ then the following conditions hold a) $(D_t^k I^k g)(t) = g(t)$, b) $(I^k D_t^k g)(t) = g(t) - \sum_{i=0}^{n-1} \frac{t^i}{i!} g^i(0)$, c) Specifically, if $0 < k < 1$, then $(I^k D_t^k g)(t) = g(t) - g(0)$, and d) For a constant function $g(t) = b$ then $D_t^k(b) = 0$.*

# 3 Limitation Of The Study

The compartmental structure simplifies the complexity of human behavior, potentially over- looking intricacies like fluctuating attitudes or demographic variability. Additionally, the homogeneous mixing assumption ignores social and geographic clustering of xenophobic be- haviors, while the fractional-order framework, though mathematically robust, may pose in- terpretability challenges for policymakers. Lastly, the optimal control strategies assume ideal conditions, neglecting practical constraints like resource limitations or political feasi- bility. Future research should validate the model with empirical data and refine its structure to enhance real-world relevance.

# 4 Model Formulation

## 4.1 Assumption And Description

In order to analyze the dynamics of public perception towards xenophobia, we divided the targeted population denoted by N(t) at a given time t to five mutually exclusive compart- ments. Individuals who are protected from xenophobic attitudes are denoted by P(t). These individuals are typically open-minded, tolerant, empathetic, and respectful towards people from different cultures, backgrounds, or nationalities. They do not harbor feelings of fear, hatred, or hostility towards individuals who are perceived as foreign or different. Instead, they embrace diversity, value inclusivity, and seek to understand and appreciate the rich- ness that different perspectives and experiences bring to society. These individuals are able to recognize the shared humanity among all people and reject discriminatory attitudes or behaviors based on nationality, ethnicity, or cultural differences. Individuals who are suscep- tible to being affected by xenophobic attitudes are denoted by S(t). These individuals may exhibit characteristics such as fear of the unknown, intolerance towards cultural differences, prejudice based on stereotypes, feelings of superiority over others, and a lack of exposure to diverse perspectives. They may be influenced by negative rhetoric or propaganda that perpetuates xenophobic attitudes, leading them to view individuals from different backgrounds as threats or outsiders. These individuals may feel insecure about their own identity or place in society, leading them to scapegoat or blame others for societal problems. Additionally, a lack of education or awareness about other cultures and histories can contribute to the development of xenophobic beliefs. Individuals who are exposed to xenophobic attitudes are denoted by E(t). These individuals may exhibit prejudice, discrimination, and hostility towards individuals or groups perceived as foreign or different. They might harbor negative attitudes and beliefs based on fear, stereotypes, or misinformation about people from other countries or cultures. Such individuals may express xenophobic views through verbal attacks, exclusionary behavior, or support for policies that limit the rights and opportunities of others. Individuals who are affected by xenophobic attitudes are denoted by A(t). These individuals often exhibit characteristics such as fear or mistrust of individuals from different cultural backgrounds, a tendency to view outsiders as a threat to their own identity or way of life, a lack of empathy or understanding towards those perceived as ”foreign” and a belief in the superiority of their own group or culture. They may also display closed-mindedness, prejudice, and a reluctance to engage with or learn about other cultures. Individuals who have recovered from xenophobic attitudes are denoted by R(t). These individuals have undergone a transformation in their attitudes and beliefs towards those they once viewed as different or foreign. They have overcome biases, prejudices, and negative stereotypes that fueled their xenophobic views. Through education, exposure to diversity, empathy-building experiences, and personal reflection, these individuals have developed a more inclusive, open-minded, and accepting perspective towards people from other cultures or backgrounds. They now show respect, appreciation, and support for diversity, and actively work towards creating a more welcoming and harmonious society for all. These state variables help in classifying individuals according to

Research Paper
Open Access
ISSN: 3108-4052

their attitudes towards xenophobia. While there may be various factors that can contribute to the development and rein- forcement of xenophobic attitudes among individuals, our focus in developing a simplified mathematical model is solely on how people's attitudes towards xenophobia evolve through their interactions. If N (t) is the population size, then

$$N(t) = P(t) + S(t) + E(t) + A(t) + R(t) \tag{4.1}$$

In order to create a model that captures the dynamics of public perception towards xenophobia in a community, it was necessary to take into account specific essential assumptions.

A fraction $\alpha$ of newly recruited individuals who are already protected, with a recruitment rate represented by $\Omega$, joined the protected group, while the number of individuals likely to engage in xenophobic attitudes increases as the remaining $(1-\alpha)\Omega$ newly recruited individuals joined.

Individuals who were previously protected and lost their protection transitioned to individuals likely to engage in xenophobic attitudes at a transfer rate of $(1-\chi)\varphi$, with $\chi$ representing the efficacy of protection.

As individuals were not permanently recovered from xenophobic attitudes, they moved from the compartment of individuals who had recovered from xenophobic attitudes to the compartment of individuals who are likely to be affected by xenophobic attitudes. This transition occurred at a specific rate denoted by the symbol $\omega$.

Individuals within the community who were vulnerable to being influenced by xenophobic attitudes would consistently be exposed to it at a rate determined by

$$\lambda(t) = \frac{\beta}{N(t)} A(t). \tag{4.2}$$

where $\beta$ is the transmission rate.

Some individuals impacted by xenophobic attitudes recovered and transitioned to compartment R at a rate of $\sigma$.

Individuals subjected to xenophobic attitudes could not engage in reflecting xenophobic attitudes and align with the compartment R at a rate of $\xi$.

In all compartments, the natural death rate is $\mu$.

Additional parameters and state variables are stated in Table 1 and Table 2 provided below.

**Table 1:** Descriptions of state variables

| Variable | Description |
|---|---|
| $P$ | The number of individuals who are protected from xenophobic attitudes. |
| $S$ | The number of individuals who are susceptible to being affected by xenophobic attitudes. |
| $E$ | The number of individuals who are exposed to xenophobic attitudes. |
| $A$ | The number of individuals who are affected by xenophobic attitudes. |
| $R$ | The number of individuals who have recovered from xenophobic attitudes. |

Research Paper
Open Access
ISSN: 3108-4052

**Table 2:** Description of model parameters

| Parameters | Description |
|---|---|
| $\alpha$ | A fraction of recruited individuals who are already protected. |
| $\varphi$ | Rate at which protected individuals lose their xenophobia protection. |
| $\omega$ | Rate of transition from recovered to potentially xenophobic state. |
| $\xi$ | Transfer rate from exposed (E) to recovered (R). |
| $\psi$ | Transfer rate from exposed (E) to affected (I). |
| $\sigma$ | Recovery rate from xenophobic attitudes. |
| $\Omega$ | Recruitment rate. |
| $\chi$ | Efficacy of protection against xenophobia. |

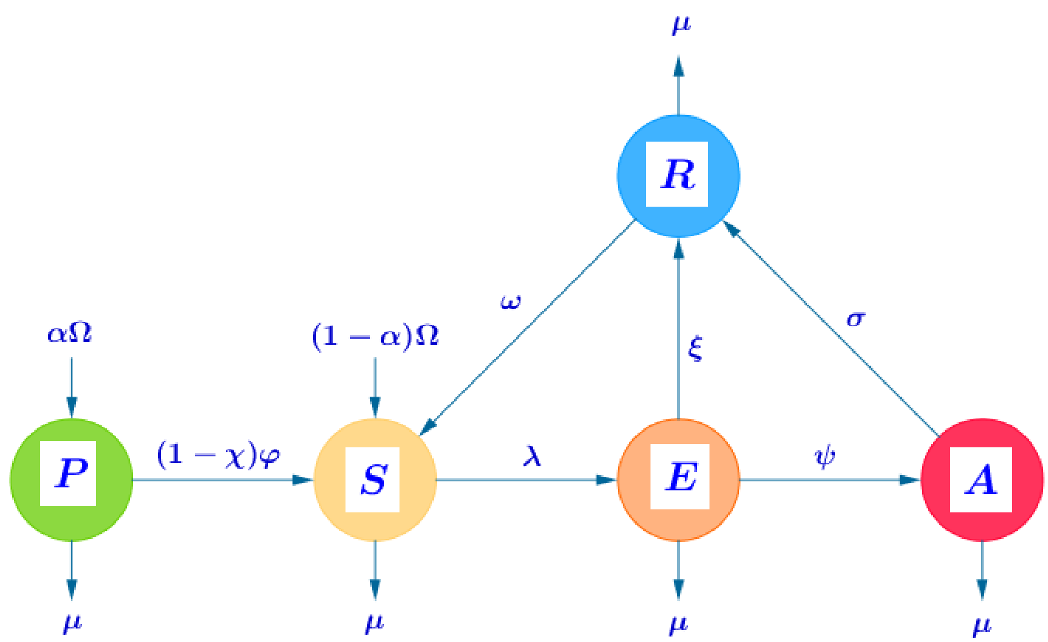


**Figure 1:** Schematic diagram for the model

## 4.2 Deterministic Integer Order Model

The interactions between the compartments are represented by the following system of non-linear deterministic equations.

$$
\begin{aligned}
\frac{dP}{dt} &= \alpha\Omega - (\mu + (1-\chi)\varphi)P \\
\frac{dS}{dt} &= (1-\alpha)\Omega + \omega R + (1-\chi)\varphi P - (\lambda + \mu)S \\
\frac{dE}{dt} &= \lambda S - (\mu + \xi + \psi)E \\
\frac{dA}{dt} &= \psi E - (\mu + \sigma)A \\
\frac{dR}{dt} &= \xi E + \sigma A - (\mu + \omega)R
\end{aligned}
\tag{4.3}
$$

Research Paper
Open Access
ISSN: 3108-4052

with initial conditions

$$P(0) \geq 0, S(0) > 0, E(0) \geq 0, A(0) \geq 0, \text{and } R(0) \tag{4.4}$$

Note that the sum of all the differential equations becomes $\frac{dN}{dt} = \Omega - \mu N$.

## 4.3 Deterministic Fractional Order Model

Ordinary differential equations of integer order that describe dynamical systems do not account for the memory effect, meaning the system's response is unaffected by previous events [18]. One approach to integrating memory influence into a mathematical model is to convert the integer order model into a fractional order model. In this section, we reformulate the traditional integer order model using the Caputo fractional order derivative. This modification enables us to incorporate memory effects and improve our understanding of the dynamics of xenophobic attitudes within a community. By utilizing the properties of fractional order derivatives and applying the Caputo fractional order derivative, we achieve

$$\begin{aligned}
{}^{C}D_t^{\kappa}P &= \alpha^{\kappa}\Omega^{\kappa} - (\mu^{\kappa} + (1-\chi^{\kappa})\varphi^{\kappa})P \\
{}^{C}D_t^{\kappa}S &= (1-\alpha^{\kappa})\Omega^{\kappa} + \omega^{\kappa}R + (1-\chi^{\kappa})\varphi^{\kappa}P - (\lambda^{\kappa} + \mu^{\kappa})S \\
{}^{C}D_t^{\kappa}E &= \lambda^{\kappa}S - (\mu^{\kappa} + \xi^{\kappa} + \psi^{\kappa})E \\
{}^{C}D_t^{\kappa}A &= \psi^{\kappa}E - (\mu^{\kappa} + \sigma^{\kappa})A \\
{}^{C}D_t^{\kappa}R &= \xi^{\kappa}E + \sigma^{\kappa}A - (\mu^{\kappa} + \omega^{\kappa})R
\end{aligned} \tag{4.5}$$

Subject to $P(0) \geq 0, S(0) > 0, E(0) \geq 0, A(0) \geq 0$, and $R(0) \geq 0$.

### 4.3.1 Feasibility of the fractional order model

**Theorem 4.1.** *Every solution of the fractional order model given in Equation* (4.5) *with the initial conditions* $P(0) \geq 0, S(0) > 0, E(0) \geq 0, A(0) \geq 0$ *and* $R(0) \geq 0$ *is non-negative.*

*Proof.* Applying the same method as described in [19], we make the assumption that the second equation of fractional order model given in Equation (4.5) is false.

Let $t_* = \min\{t : P(t)S(t)E(t)A(t)R(t) = 0\}$. If $S(t_*) = 0$, then $P(t) \geq 0, E(t) \geq 0, A(t) \geq 0$ and $R(t) \geq 0$ for all $0 \leq t \leq t_*$.

Let us assume that

$$A_1 = \min_{0 \leq t \leq t_*} \left\{ \frac{1}{S}\left((1-\alpha^{\kappa})\Omega^{\kappa} + \omega^{\kappa}R + (1-\chi^{\kappa})\varphi^{\kappa}P\right) - (\lambda^{\kappa} + \mu^{\kappa}) \right\}$$

and it exists. Consequently,

$${}^{C}D_t^{\kappa}S(t) - A_1 S(t) \geq 0.$$

Using the Laplace transform, we obtain the following

$$s^{\kappa}\tilde{S}(s) - s^{\kappa-1}S(0) - A_1\tilde{S}(s) \geq 0.$$

Thus, $\tilde{S}(s) \geq S(0)\frac{s^{\kappa-1}}{s^{\kappa}-A_1} = \frac{S(0)}{s}\left(1 - \frac{A_1}{s^{\kappa}}\right)^{-1} = S(0)\sum_{i=0}^{\infty}\frac{A_1^i}{s^{i\kappa+1}}$. Applying the inverse Laplace transform, we obtain

$$S(t) \geq S(0)\sum_{i=0}^{\infty}\frac{(A_1 t^{\kappa})^i}{\Gamma(i\kappa+1)} = S(0)E_{\kappa}(A_1 t^{\kappa}) > 0,$$

which contradicts the assumption $S(t_*) = 0$ for any finite time $t_*$.

Furthermore, analyzing the first equation of system (4.5) yields:

$$^{C}D_t^{\kappa} P \geq -(\mu^{\kappa} + (1 - \chi^{\kappa})\varphi^{\kappa})P,$$

implying

$$P(t) \geq P(0)e^{-(\mu^{\kappa}+(1-\chi^{\kappa})\varphi^{\kappa})t} \geq 0.$$

By the same analogy, we can see that each solution of the fractional order model Equation (4.5) is non-negative. □

Research Paper
Open Access
ISSN: 3108-4052

**Theorem 4.2.** *Each solution of the fractional order model given in Equation* (4.5) *is bounded in the region* $\Omega = \{(P,S,E,A,R) \in \mathbb{R}^5_+ : N(t) \leq \frac{\Omega^\kappa}{\mu^\kappa}\}$.

*Proof.* ${}^{C}D^\kappa_t N(t) = D^\kappa_t P + D^\kappa_t S + D^\kappa_t E + D^\kappa_t A + D^\kappa_t R = \Omega^\kappa - \mu^\kappa N(t)$. Employing the Laplace transform to both sides of the equation, we get $\mathcal{L}(D^\kappa_t N(t)) = \mathcal{L}(\Omega^\kappa - \mu^\kappa N(t)) = \frac{\Omega^\kappa}{s} - \mu^\kappa \mathcal{L}(N(t))$. After reorganizing and simplifying the equation, we arrive at

$$\mathcal{L}(N(t)) = \frac{\Omega^\kappa s^{-1}}{s^\kappa + \mu^\kappa} + \frac{s^{\kappa-1}}{s^\kappa + \mu^\kappa} N(0).$$

Next, we apply the inverse Laplace transform and utilize a specific property of the Laplace transform of the Mittag Leffler function, as referenced in Proposition 2.7. This allows us to express $N(t)$ in the time domain:

$$N(t) = N(0)E_\kappa(-\mu^\kappa t^\kappa) + \frac{\Omega^\kappa}{\mu^\kappa}(1 - E_\kappa(-\mu^\kappa t^\kappa)).$$

As $t$ approaches infinity, the term $E_\kappa(-\mu^\kappa t^\kappa)$ converges to zero, leading us to the conclusion that $N(t) \to \frac{\Omega^\kappa}{\mu^\kappa}$. Now, we consider the behavior of the population $N(t)$. If $N(0) \leq \frac{\Omega^\kappa}{\mu^\kappa}$, then for every $t \geq 0 : 0 < N(t) \leq \frac{\Omega^\kappa}{\mu^\kappa}$. □

This indicates that the total number of individuals $N(t)$ is bounded above by $\frac{\Omega^\kappa}{\mu^\kappa}$. Finally, we analyze the feasible region defined as $\{(P,S,E,A,R) \in \mathbb{R}^5 : P+S+E+A+R \leq \frac{\Omega^\kappa}{\mu^\kappa}\}$ is positively invariant. Therefore, the formulation presented in Equation (4.5) is not only well-defined but also holds mathematical, biological, and epidemical validity within this bounded region.

### 4.3.2 Qualitative analysis

**Local stability of xenophobia-free equilibrium point.** The xenophobic attitudes-free equilibrium point in the fractional-order model denotes a condition in which xenophobic attitudes are either minimal or entirely absent. This equilibrium reflects a society in which xenophobia is effectively suppressed and prevented from proliferating. In contrast, the xenophobic attitudes-persistent equilibrium point indicates a state where the prevalence of xenophobic attitudes stabilizes at a certain level within a given population or society, implying a scenario where these attitudes continue to spread over time. To identify the xenophobic attitudes-free equilibrium point in the fractional-order model, we begin by setting:

$$CD^\kappa_t P(t) = 0,\ D^\kappa_t S(t) = 0,\ D^\kappa_t E(t) = 0,\ D^\kappa_t A(t) = 0,\ D^\kappa_t R(t) = 0.$$

Thus, the equilibrium point characterized by the absence of xenophobic attitudes is

$$E^0_x = (P^0, S^0, E^0, A^0, R^0) = \left(\frac{\alpha^\kappa \Omega^\kappa}{\mu^\kappa + (1-\chi^\kappa)\varphi^\kappa}, \frac{\Omega^\kappa[(1-\alpha^\kappa)\mu^\kappa + (1-\chi^\kappa)\varphi^\kappa]}{(\lambda^\kappa + \mu^\kappa)(\mu^\kappa + (1-\chi^\kappa)\varphi^\kappa)}, 0, 0, 0\right)$$

The next generation matrix approach as utilized by [20], is applied here to calculate the effective reproduction number, denoted as $\mathcal{R}^\kappa_{eff}$, for the spread of xenophobic attitudes. This effective reproduction number quantifies the average number of individuals who develop xenophobic attitudes as a consequence of one person's engagement in spreading these attitudes within a community, particularly when various control strategies—such as law enforcement, judicial systems, and socio-economic interventions addressing issues like, poverty, unemployment, and inequality are implemented. The determination of $\mathcal{R}^\kappa_{eff}$ relies on the spectral radius, which corresponds to the dominant eigenvalue

Research Paper
Open Access
ISSN: 3108-4052

of the matrix $FV^{-1} = \left(\frac{\partial \mathcal{F}_i(E_x^0)}{\partial y_j}\right)\left(\frac{\partial \nu_i(E_x^0)}{\partial y_j}\right)^{-1}$, where $\mathcal{F}_i$ is the rate at which newly individuals who were engaged in disseminating xenophobic attitudes in compartment $i$, $\nu_i$ is the transfer of individuals who developed xenophobic attitudes from existing compartment $i$ to another compartment and $E_x^0$ is the xenophobic attitudes free equilibrium point. Let $y = (E_g, A_g)^T$. Then, $\frac{dy}{dt} = F - V$.

The xenophobic attitudes compartments are E and A.

$$ {}^C D_t^\kappa E = \lambda^\kappa S - (\mu^\kappa + \xi^\kappa + \psi^\kappa)E $$
$$ {}^C D_t^\kappa A = \psi^\kappa E - (\mu^\kappa + \sigma^\kappa)A $$

$$ \mathcal{F} = \begin{pmatrix} \lambda^\kappa S \\ 0 \end{pmatrix} \text{ and } \mathcal{V} = \begin{pmatrix} -(\mu^\kappa + \xi^\kappa + \psi^\kappa)E \\ \psi^\kappa E - (\mu^\kappa + \sigma^\kappa)A \end{pmatrix} $$

Let $f_1(E, A) = \lambda^\kappa S = \frac{\beta^\kappa}{N} AS$ and $f_2(E, A) = 0$

$$ F = \begin{pmatrix} \frac{\partial f_1}{\partial E} & \frac{\partial f_1}{\partial A} \\ \frac{\partial f_2}{\partial E} & \frac{\partial f_2}{\partial A} \end{pmatrix}_{(P^0,S^0,E^0,A^0,R^0)} = \begin{pmatrix} 0 & \frac{\beta^\kappa S^0}{P^0+S^0} \\ 0 & 0 \end{pmatrix} $$

Let $v_1(E, A) = -(\mu^\kappa + \xi^\kappa + \psi^\kappa)E$ and $v_2(E, A) = \psi^\kappa E - (\mu^\kappa + \sigma^\kappa)A$

$$ V = \begin{pmatrix} \frac{\partial v_1}{\partial E} & \frac{\partial v_1}{\partial A} \\ \frac{\partial v_2}{\partial E} & \frac{\partial v_2}{\partial A} \end{pmatrix}_{(P^0,S^0,E^0,A^0,R^0)} = \begin{pmatrix} -(\mu^\kappa + \xi^\kappa + \psi^\kappa) & 0 \\ \psi^\kappa & -(\mu^\kappa + \sigma^\kappa) \end{pmatrix} $$

To find the effective reproduction number $\mathcal{R}_{eff}^\kappa$, we need to compute the spectral radius of the matrix $FV^{-1}$, where

$$ F = \begin{bmatrix} 0 & \beta^\kappa \frac{S^0}{P^0+S^0} \\ 0 & 0 \end{bmatrix}, \text{ and } V = \begin{bmatrix} -(\mu^\kappa + \xi^\kappa + \psi^\kappa) & 0 \\ \psi^\kappa & -(\mu^\kappa + \sigma^\kappa) \end{bmatrix}. $$

Then

$$ FV^{-1} = \begin{pmatrix} -\frac{\beta^\kappa \psi^\kappa S^0}{(P^0+S^0)(\mu^\kappa+\xi^\kappa+\psi^\kappa)(\mu^\kappa+\sigma^\kappa)} & -\frac{\beta^\kappa S^0}{(P^0+S^0)(\mu^\kappa+\sigma^\kappa)} \\ 0 & 0 \end{pmatrix} $$

The effective reproduction number $\mathcal{R}_{eff}^\kappa = \rho(FV^{-1})$ where $\rho(\cdot)$ denotes the spectral radius is given by

$$ \mathcal{R}_{eff}^\kappa = \frac{\beta^\kappa \psi^\kappa [(1-\alpha^\kappa)\mu^\kappa + (1-\chi^\kappa)\varphi^\kappa(1+\alpha^\kappa)]}{[(1-\alpha^\kappa)\mu^\kappa + (1-\chi^\kappa)\varphi^\kappa(1+\alpha^\kappa)](\mu^\kappa + (1-\chi^\kappa)\varphi^\kappa) + \alpha^\kappa(\lambda^\kappa + \mu^\kappa)} $$

**Theorem 4.3.** *Consider a system of fractional order differential equations given by $D_0^\kappa X(t) = f(X(t)), 0 < \kappa \leq 1$, where $D_0^\kappa$ denotes the fractional derivative of order $\kappa$. Let $B = D(X_0)$ represent the Jacobean matrix of $f$ evaluated at the equilibrium point $X_0$. The equilibrium point $X_0$ is locally and asymptotically stable if and only if, for each eigenvalue $\lambda_i$ of the matrix $B$, $|\arg(\lambda_i)| > \frac{\kappa\pi}{2}$, where $\arg(\lambda_i)$ denotes the argument (or phase) of the eigenvalue of $\lambda_i$. This stability criterion is established in the work of [16].*

In simpler terms, the equilibrium point $X_0$ is stable if all eigenvalues of the Jacobian matrix $B$ lie in a region of the complex plane where their arguments are greater than $\frac{\kappa\pi}{2}$. This ensures that the system's solutions decay to the equilibrium point over time, guaranteeing both local and asymptotic stability.

**Theorem 4.4.** *The xenophobia-free equilibrium point $E^0$ of the model represented by Equation (4.5) is both locally and asymptotically stable when $\mathcal{R}_{eff}^\kappa < 1$. Otherwise, the equilibrium point is unstable.*

Research Paper
Open Access
ISSN: 3108-4052

*Proof.* At the xenophobia-free equilibrium point, the Jacobian matrix for the dynamical system described by Equation (4.5) is expressed as follows

$$J(E_x^0) = \begin{pmatrix} -(\mu^\kappa + (1-\chi^\kappa)\varphi^\kappa) & 0 & 0 & 0 & 0 \\ (1-\chi^\kappa)\varphi^\kappa & -\mu^\kappa & 0 & -\beta^\kappa \frac{S^0}{P^0+S^0} & \omega^\kappa \\ 0 & 0 & -(\mu^\kappa + \xi^\kappa + \psi^\kappa) & \beta^\kappa \frac{S^0}{P^0+S^0} & 0 \\ 0 & 0 & \psi^\kappa & -(\mu^\kappa + \sigma^\kappa) & 0 \\ 0 & 0 & \xi^\kappa & \sigma^\kappa & -(\mu^\kappa + \omega^\kappa) \end{pmatrix}$$

To compute the eigenvalues of the Jacobian matrix, we solve the characteristic equation

$$\det(J(E_x^0) - \lambda I) = 0$$

where $\lambda$ is an eigenvalue and $I$ is the $5 \times 5$ identity matrix.

$$\det(J(E_x^0) - \lambda I) = (-(\mu^\kappa + (1-\chi^\kappa)\varphi^\kappa) - \lambda)(-\mu^\kappa - \lambda)(-(\mu^\kappa + \omega^\kappa) - \lambda)\,(-(\mu^\kappa + \xi^\kappa + \psi^\kappa) - \lambda)\,(-(\mu^\kappa + \sigma^\kappa) - \lambda) - \beta^\kappa \frac{S^0}{P^0 + S^0}\psi^\kappa).$$

This gives the eigenvalues: $\lambda_1 = -(\mu^\kappa + (1-\chi^\kappa)\varphi^\kappa), \lambda_2 = -\mu^\kappa, \lambda_3 = -(\mu^\kappa + \omega^\kappa)$. The remaining eigenvalues $\lambda_{4,5}$ satisfy:

$$\left(-(\mu^\kappa + \xi^\kappa + \psi^\kappa) - \lambda)(-(\mu^\kappa + \sigma^\kappa) - \lambda) - \beta^\kappa \frac{S^0}{P^0 + S^0}\psi^\kappa\right) = 0$$

In the standard form, this becomes

$$a\lambda^2 + b\lambda + c = 0$$

where $a = 1, b = 2\mu^\kappa + \xi^\kappa + \psi^\kappa + \sigma^\kappa$, and $c = (\mu^\kappa + \xi^\kappa + \psi^\kappa)(\mu^\kappa + \sigma^\kappa)(1 - \mathcal{R}_{eff}^\kappa)$.

This confirms that when the coefficients $a, b$ and $c$ of the characteristic equation are all positive and the effective reproduction number satisfies $\mathcal{R}_{eff}^\kappa < 1$, the eigenvalues will have negative real parts. Furthermore, by applying Theorem 4.3 which requires $|\arg(\lambda_i)| > \frac{\kappa\pi}{2}$ for each eigenvalue $\lambda_i$ ($i = 1, 2, 3, 4,$ and $5$) and fractional order $0 < \kappa \leq 1$, we conclude that the xenophobia free equilibrium point of the model (given by Equation (4.5)) is locally asymptotically stable. ☐

**Xenophobia-persistent equilibrium point and bifurcation analysis.** The xenophobia-persistent equilibrium point represents a state in which xenophobic attitudes remain entrenched within the population. To determine this equilibrium for the Caputo fractional-order model (given in Equation (4.5)), we set the right-hand side of all system equations to zero and solve for the non-trivial steady-state solution. Let $E_x^* = (P^*, S^*, E^*, A^*, R^*)$ represent the xenophobia persistent equilibrium point of the system

$$\begin{aligned} {}^C D_t^\kappa P^* &= \alpha^\kappa \Omega^\kappa - (\mu^\kappa + (1-\chi^\kappa)\varphi^\kappa)P^* = 0 \\ {}^C D_t^\kappa S^* &= (1-\alpha^\kappa)\Omega^\kappa + \omega^\kappa R^* + (1-\chi^\kappa)\varphi^\kappa P^* - (\lambda^\kappa + \mu^\kappa)S^* = 0 \\ {}^C D_t^\kappa E^* &= \lambda^\kappa S^* - (\mu^\kappa + \xi^\kappa + \psi^\kappa)E^* = 0 \\ {}^C D_t^\kappa A^* &= \psi^\kappa E^* - (\mu^\kappa + \sigma^\kappa)A^* = 0 \\ {}^C D_t^\kappa R^* &= \xi^\kappa E^* + \sigma^\kappa A^* - (\mu^\kappa + \omega^\kappa)R^* = 0 \end{aligned} \tag{4.6}$$

Research Paper
Open Access
ISSN: 3108-4052

Upon solving the system given in Equation (4.6), we obtain the xenophobia-persistent equilibrium state, expressed as:

$$\begin{aligned} E_x^* &= (P^*, S^*, E^*, A^*, R^*) \\ &= \Bigg( \frac{\alpha^\kappa \Omega^\kappa}{\mu^\kappa + (1-\chi^\kappa)\varphi^\kappa}, \\ &\quad \frac{(1-\alpha^\kappa)(\mu^\kappa + (1-\chi^\kappa)\varphi^\kappa)\Omega^\kappa + (1-\chi^\kappa)\varphi^\kappa \alpha^\kappa \Omega^\kappa}{(\mu^\kappa + (1-\chi^\kappa)\varphi^\kappa)(\lambda^\kappa + \mu^\kappa - \omega^\kappa \mathcal{C})}, \\ &\quad \frac{\lambda^\kappa S^*}{\mu^\kappa + \xi^\kappa + \psi^\kappa}, \frac{\psi^\kappa \lambda^\kappa S^*}{(\mu^\kappa + \sigma^\kappa)(\mu^\kappa + \xi^\kappa + \psi^\kappa)}, \\ &\quad \frac{\lambda^\kappa S^* (\xi^\kappa(\mu^\kappa + \sigma^\kappa) + \sigma^\kappa \psi^\kappa)}{(\mu^\kappa + \omega^\kappa)(\mu^\kappa + \sigma^\kappa)(\mu^\kappa + \xi^\kappa + \psi^\kappa)} \Bigg). \end{aligned} \tag{4.7}$$

**Theorem 4.5.** *The dynamical model in Equation* (4.5) *which describes how public perception of xenophobia evolves in a community, has a long term equilibrium state when* $\mathcal{R}_{eff}^\kappa > 1$, *provided that either* $\mathbf{a}_1 > 0$, *or* $\mathbf{a}_1 < 0$. *However, if* $\mathcal{R}_{eff}^\kappa < 1$ *and* $\mathbf{a}_1 < 0$, *the system may exhibit two possible persistent equilibrium points.*

**Theorem 4.6.** *The xenophobia-persistent equilibrium point* $E^*$, *defined by the coordinates in Equation* (4.7), *is locally asymptotically stable for the model given in Equation* (4.5).

*Proof.* The proofs of Theorems 4.5 and 4.6 proceed analogously to the method presented in [21–23] adapting the framework to our system. □

# 5 Optimal Control Problem

In this study, we extend the model from Equation (4.5) by introducing two time-dependent control measures:

- $w_1(t)$: Efforts to tackle entrenched socio-economic development issues, including poverty, unemployment, and inequality.
- $w_2(t)$: Initiatives to strengthen law enforcement and improve judicial efficiency.

Both controls are bounded, with $0 \leq w_1(t), w_2(t) \leq 1$.

The protective measures under $w_1(t)$ reduce the number of susceptible individuals exposed to risk by a factor of $(1 - w_1(t))$. Similarly, interventions under $w_2(t)$—such as cultural education, addressing underlying fears, and promoting inclusivity—diminish the population affected by xenophobic attitudes by $(1 - w_2(t))$.

The updated fractional-order model with time-dependent controls is given

$$\begin{aligned} {}^C D_t^\kappa P &= \alpha^\kappa \Omega^\kappa - (\mu^\kappa + (1-\chi^\kappa)\varphi^\kappa) P \\ {}^C D_t^\kappa S &= (1-\alpha^\kappa)\Omega^\kappa + \omega^\kappa R + (1-\chi^\kappa)\varphi^\kappa P - ((1-w_1(t))\lambda^\kappa + \mu^\kappa) S \\ {}^C D_t^\kappa E &= (1-w_1(t))\lambda^\kappa S - (\mu^\kappa + \xi^\kappa + \psi^\kappa) E \\ {}^C D_t^\kappa A &= \psi^\kappa E - (\mu^\kappa + w_2 + \sigma^\kappa) A \\ {}^C D_t^\kappa R &= \xi^\kappa E + (w_2 + \sigma^\kappa) A - (\mu^\kappa + \omega^\kappa) R \end{aligned} \tag{5.1}$$

Research Paper
Open Access
ISSN: 3108-4052

The system evolves from non-negative initial states: $P(0) \geq 0, S(0) > 0, E(0) \geq 0, A(0) \geq 0, R(0) \geq 0$. The admissible control set is defined as:

$$\Omega_c = \{(w_1(t), w_2(t)) : 0 \leq w_1(t), w_2(t) \leq 1, t \in [0, T]\} \text{ where } T \text{ denotes the intervention period.}$$

Interpretation of control Extremes:

- If $w_1(t) = 0$ and $w_2(t) = 0$, no additional interventions are applied to curb xenophobic attitudes.
- If $w_1(t) = 1$ and $w_2(t) = 1$, the interventions are assumed to be fully (100%) effective—although this assumption does not align with reality.

To reduce the number of individuals developing xenophobic attitudes in the community, we formulate the objective function:

$$J(w_1, w_2) = \int_0^T \left( \Pi_1 E + \Pi_2 A + \frac{\$_1}{2} w_1^2 + \frac{\$_2}{2} w_2^2 \right) dt \tag{5.2}$$

Parameters:

- $T$ is the final time of intervention implementation
- $\Pi_1$ and $\Pi_2$: Positive weighting constants corresponding to $E$ (exposed) and $A$ (active xenophobic individuals).
- $\$_1$ and $\$_2$: Cost coefficients for implementing controls $w_1$ (socio-economic interventions) and and $w_2$ (law enforcement/cultural education), respectively.

Additionally, this ensures unit consistency and reflects the nonlinear costs of intervention efforts. The goal is to find optimal strategies $w^* = (w_1^*, w_2^*)$ that:

1. Minimize the number of individuals adopting xenophobic attitudes ($E$ and $A$).
2. Reduce the economic burden of implementing controls $w_1(t)$ and $w_2(t)$.
3. Satisfy the dynamical constraints in Equation (4.5).

In Equation (5.2), the term $\Pi_1 E$ denotes the cost associated with minimizing the number of individuals exposed to xenophobic attitudes, whereas $\Pi_2 M$ represents the cost of managing the group impacted by such attitudes.

**Theorem 5.1.** *There exists an optimal control $w^* = (w_1^*, w_2^*)$ within the domain $\Omega_c$, along with a solution vector $(S^*, P^*, E^*, A^*, R^*)$ for the dynamical system given in Equation* (5.1)*, under the provided initial conditions, such that $J(w_1^*, w_2^*) = \min_\Omega J(w_1, w_2)$.*

*Proof.* The proof follows a similar approach as in [24], and thus we omit the details here. □

Following Pontryagin's maximum principle (as described in [14, 25, 26], we derive the necessary conditions for the optimal control model in Equation (5.1). The Hamiltonian (H) is defined as:

$$\mathcal{H} = \Pi_1 E + \Pi_2 A + \frac{C_1}{2} w_1^2 + \frac{C_2}{2} w_2^2 \tag{5.3}$$

Here, $\lambda_1(t)$, $\lambda_2(t)$, $\lambda_3(t)$, $\lambda_4(t)$ and $\lambda_5(t)$ are the co-state (or adjoint) variables. Using the methodology from [14], we derive the following results for the fractional-order model.

Research Paper
Open Access
ISSN: 3108-4052

The transversality conditions are given by

$$\lambda_i^*(T) = 0, i = 1, 2, ..., 5.$$

**Theorem 5.2.** *The optimal control pair* $(w_1^*, w_2^*)$ *which minimizes* $J(w_1, w_2)$ *over the domain* $\Omega_c$ *is determined by:*

$$w_1^*(t) = \max\left\{0, \min\left(1, \frac{(\lambda_3 E - \lambda_1)\lambda S}{2C_1}\right)\right\},$$
$$w_2^*(t) = \max\left\{0, \min\left(1, \frac{(\lambda_3 - \lambda_4)[\sigma^\kappa \Omega^\kappa E + \gamma^\kappa M]}{2C_2}\right)\right\},$$

*where* $\lambda_1(t), \lambda_2(t), \lambda_3(t), \lambda_4(t)$ *and* $\lambda_5(t)$ *are the co-state variables satisfying Equation along with the transversality conditions stated earlier.*

*The optimal control are then expressed as:*

$$w_i^* = \begin{cases} 0, & w_i \leq 0 \\ w_i, & 0 < w_i < 1 \quad \text{for } i = 1, 2 \\ 1, & w_i \geq 1 \end{cases}$$

*Proof.* Applying Pontryagin's maximal principle (as in (Baba and Bilgehan, 2021; Teklu and Terefe, 2022)), we derive the following adjoint equations:

$$^C D_t^\kappa \lambda_1 = \frac{\partial \mathcal{H}}{\partial S},$$
$$^C D_t^\kappa \lambda_2 = \frac{\partial \mathcal{H}}{\partial P},$$
$$^C D_t^\kappa \lambda_3 = \frac{\partial \mathcal{H}}{\partial E},$$
$$^C D_t^\kappa \lambda_4 = \frac{\partial \mathcal{H}}{\partial A},$$
$$^C D_t^\kappa \lambda_5 = \frac{\partial \mathcal{H}}{\partial R},$$

subject to the transversality conditions $\lambda_i^*(T) = 0$, $i = 1, 2, \ldots, 5$.

The optimality conditions is obtained by setting the partial derivatives of the Hamiltonian $\mathcal{H}$ with respect to the control variables $c_1$ and $c_2$ to zero:

$$\frac{\partial \mathcal{H}}{\partial c_1} = 0 \text{ and } \frac{\partial \mathcal{H}}{\partial c_2} = 0.$$

□

## 5.1 Numerical Simulations

In this part, we conducted a numerical simulation to validate the analytical findings obtained from the computational model representing the public perception on xenophobic attitudes. To substantiate the results derived from the analytical computation of the model using the Caputo fractional order derivative approach and implementing the Euler forward method, we specifically developed a MATLAB code for the fractional order differential equations (FODEs) as depicted in Equation (4.5).

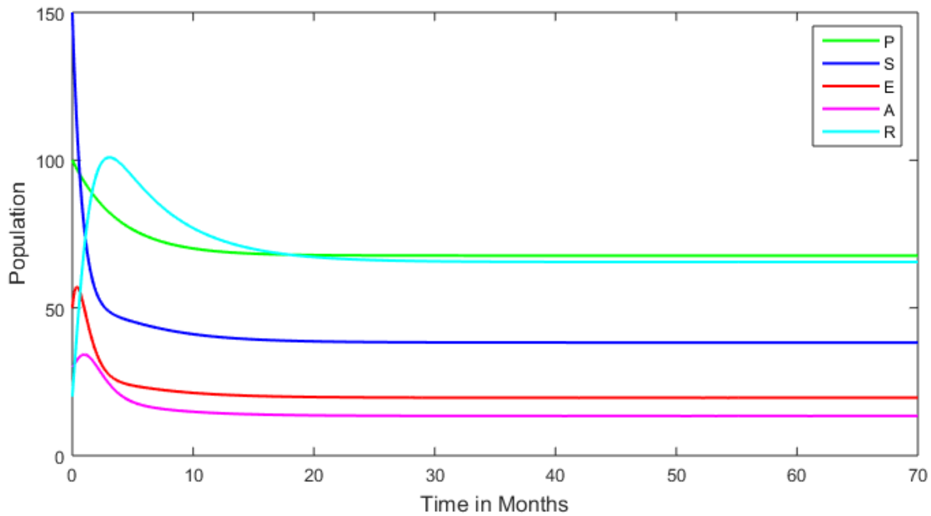


**Figure 2:** Portraits of model solutions over time

We conducted numerical simulations of the model given in Equation (4.5) in order to validate the analytical findings presented in developing the fractional order model using the ODE 45 solver MATLAB software using the baseline parameter values $\Omega = 200, \xi = 0.5, \psi = 0.5, \mu = 0.1, \alpha = 0.3, \sigma = 0.6, \omega = 0.1, \varphi = 0.1, \lambda = 0.8, \chi = 0.6,$ and $\kappa = 0.7$. Furthermore, we determined the effective reproduction number $\mathcal{R}^{\kappa}_{eff}$ of the model given by Equation (4.5) to be 2.06, signifying the persistent nature of xenophobic attitudes within the community. This calculation provided insight into the existence of xenophobic attitudes in the community, emphasizing its enduring impact.

Figure 2 illustrates the stabilization of xenophobia dynamics over time, demonstrating convergence toward a persistent equilibrium where xenophobic attitudes maintain a steady presence in the community. The sustained, non-zero trajectories indicate that the effective reproduction number ($\mathcal{R}^{\kappa}_{eff}$) exceeds one ($\mathcal{R}^{\kappa}_{eff} = 2.06 > 1$), signifying that each individual influenced by xenophobia transmits these attitudes to more than one other person on average. This perpetuates a stable endemic state, ensuring xenophobia's continued entrenchment. To shift the system toward a xenophobia-free equilibrium, targeted interventions—such as educational programs, inclusive policies, or community engagement initiatives—must reduce $\mathcal{R}^{\kappa}_{eff}$ below unity, thereby disrupting the self-sustaining cycle of xenophobic transmission. The observed stabilization underscores the resilience of xenophobic attitudes without systemic intervention.

## 5.2 Effect Of Optimal Control Strategies

The plot in Figure 3 illustrates the temporal evolution of the susceptible population's size over a 50-month period in response to socio-economic interventions. Initially, the number of individuals vulnerable to xenophobic attitudes remains high, reflecting baseline conditions where systemic issues like poverty and inequality perpetuate susceptibility. As targeted strategies (e.g., job creation, education programs, equitable resource distribution) are implemented, the curve demonstrates a gradual decline in the susceptible population. This reduction signifies that addressing root socio-economic determinants effectively diminishes the pool of individuals prone to xenophobic influences. The trend stabilizes at a lower level over time, suggesting that sustained interventions are critical to achieving long-term resilience against xenophobia. The plot underscores the importance of holistic development policies in disrupting the social drivers of xenophobic attitudes.

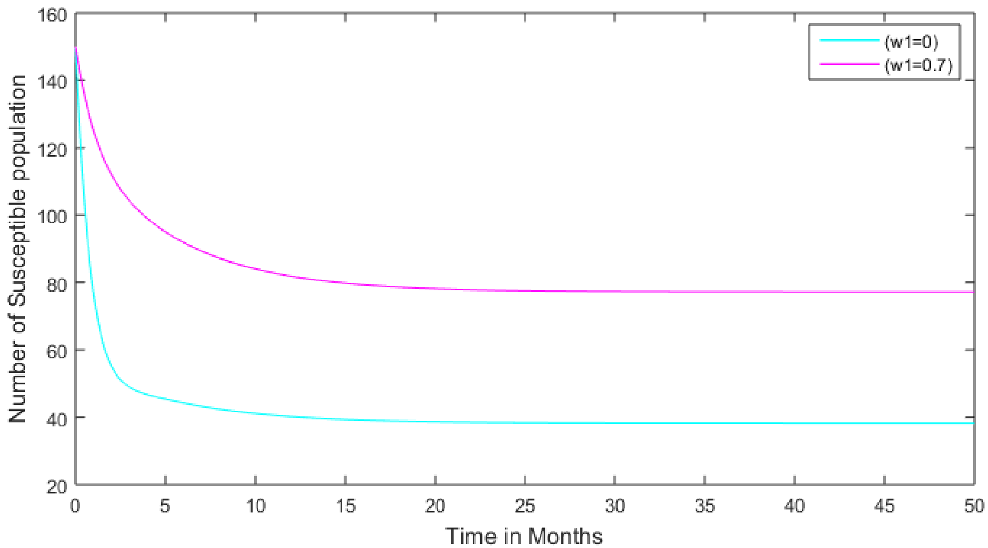


**Figure 3:** The effect of implementing strategies to tackle entrenched socio-economic development issues, including poverty, unemployment, and inequality on individuals susceptible to xenophobic attitudes

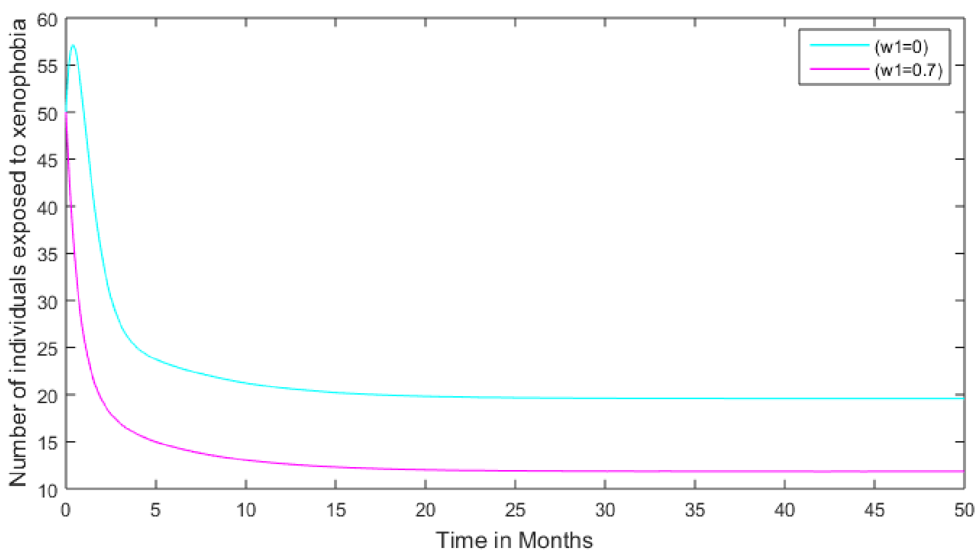


**Figure 4:** The effect of implementing strategies to tackle entrenched socio-economic development issues, including poverty, unemployment, and inequality on individuals exposed to xenophobia

The plot in Figure 4 tracks the number of individuals exposed to xenophobic attitudes over a 50-month period following the implementation of socio-economic interventions. Initially, exposure levels remain high, reflecting baseline conditions where systemic inequalities create fertile ground for xenophobia. As targeted strategies (job creation programs, educational reforms, and wealth redistribution policies) take effect, the curve shows a progressive decline in xenophobia exposure. While the curve flattens in later months, it maintains a downward trajectory, suggesting these interventions provide lasting protection against xenophobia. The plot underscores how comprehensive socio-economic development serves as both a preventive measure and long-term solution to xenophobic exposure in communities.

Research Paper
Open Access
ISSN: 3108-4052

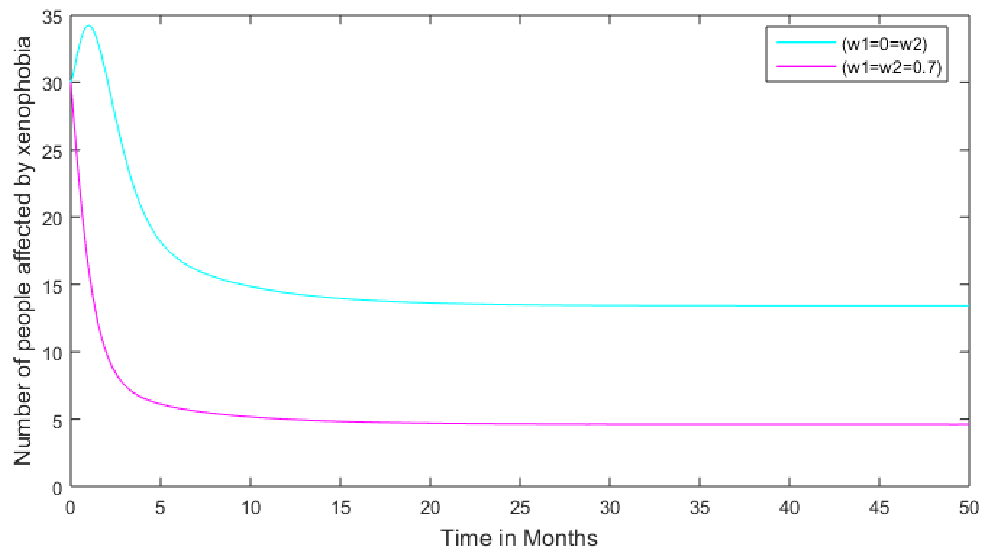


**Figure 5:** The effect of implementing strategies to tackle entrenched socio-economic development issues, including poverty, unemployment, and inequality on individuals affected by xenophobia

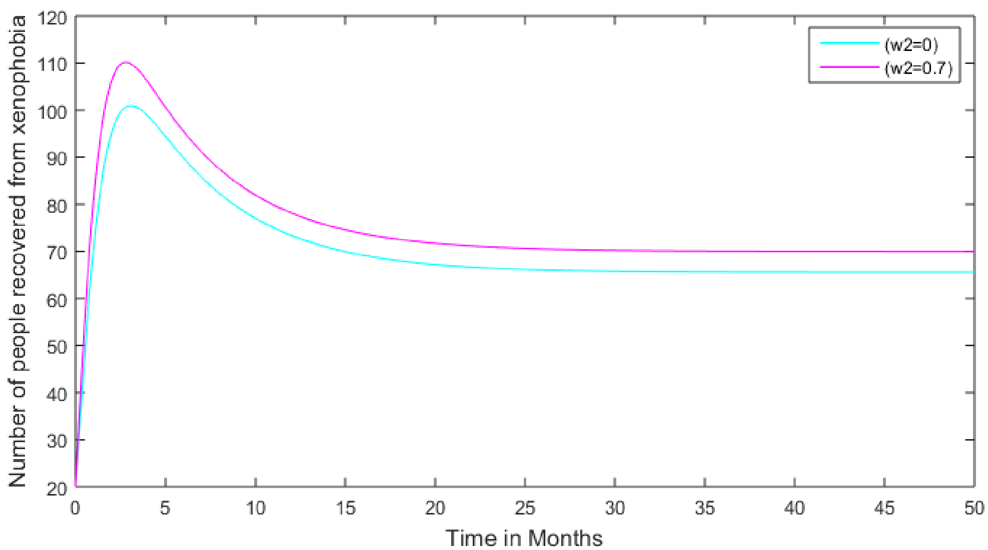


**Figure 6:** The effect of initiatives to strengthen law enforcement and improve judicial efficiency on individuals recovered from xenophobia

The plot in Figure 5 compares the prevalence of xenophobia over a 50-month period under two scenarios: absence of socio-economic interventions ($w_1 = 0$) and active implementation of such strategies ($w_1 = 0.7$). When no interventions are applied, the number of individuals affected by xenophobia remains persistently high, reflecting how systemic inequalities perpetuate xenophobic attitudes. In contrast, with targeted socio-economic measures, the curve shows a steady decline, demonstrating that addressing poverty, unemployment, and inequality effectively reduces xenophobia's impact. The gradual convergence of the two trajectories suggests that while interventions yield significant improvements, long-term commitment is required to sustain these gains. This visualization highlights the critical role of socio-economic development in mitigating xenophobia and fostering inclusive communities.

Research Paper
Open Access
ISSN: 3108-4052

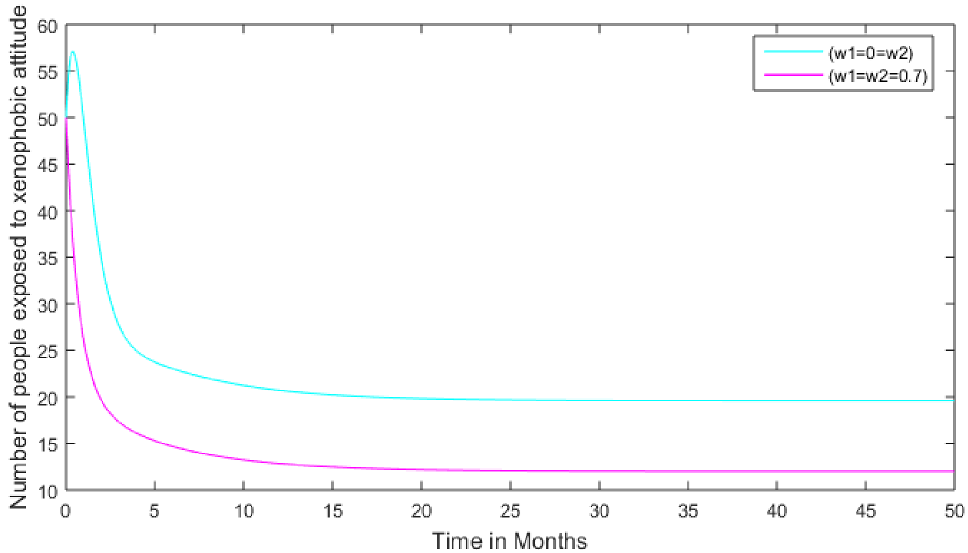


**Figure 7:** Dual intervention effect on individuals exposed to xenophobic attitudes

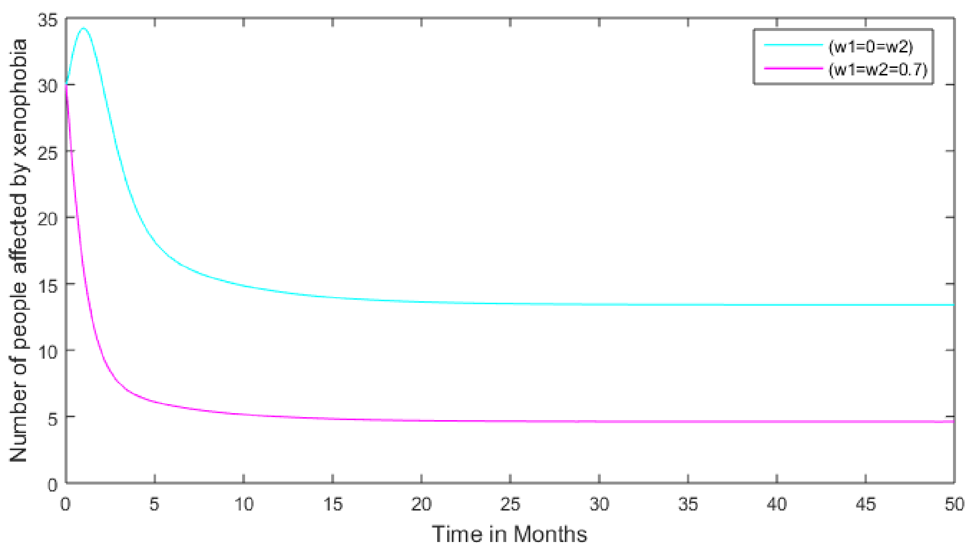


**Figure 8:** Dual intervention effect on individuals affected by xenophobia

The plot in Figure 6 tracks the cumulative number of individuals recovering from xenophobic attitudes over a 50-month period following the implementation of enhanced law enforcement measures and judicial reforms. Initially, recovery rates remain modest, reflecting the time needed for policy changes to take effect. However, after approximately 10-15 months, the curve begins a steady upward trajectory, demonstrating how improved legal frameworks and more efficient law enforcement contribute to rehabilitation from xenophobic mindsets. The acceleration in recovery rates between months 20-40 suggests that as judicial systems become more responsive to xenophobia-motivated incidents and law enforcement gains community trust, individuals increasingly disengage from xenophobic beliefs. The plot ultimately plateaus near the 50-month mark, indicating that while legal interventions produce significant recoveries, their maximum impact may require complementary social programs to address remaining cases. This visualization highlights law enforcement and judicial efficiency as critical components in societal recovery from xenophobia.

Research Paper
Open Access
ISSN: 3108-4052

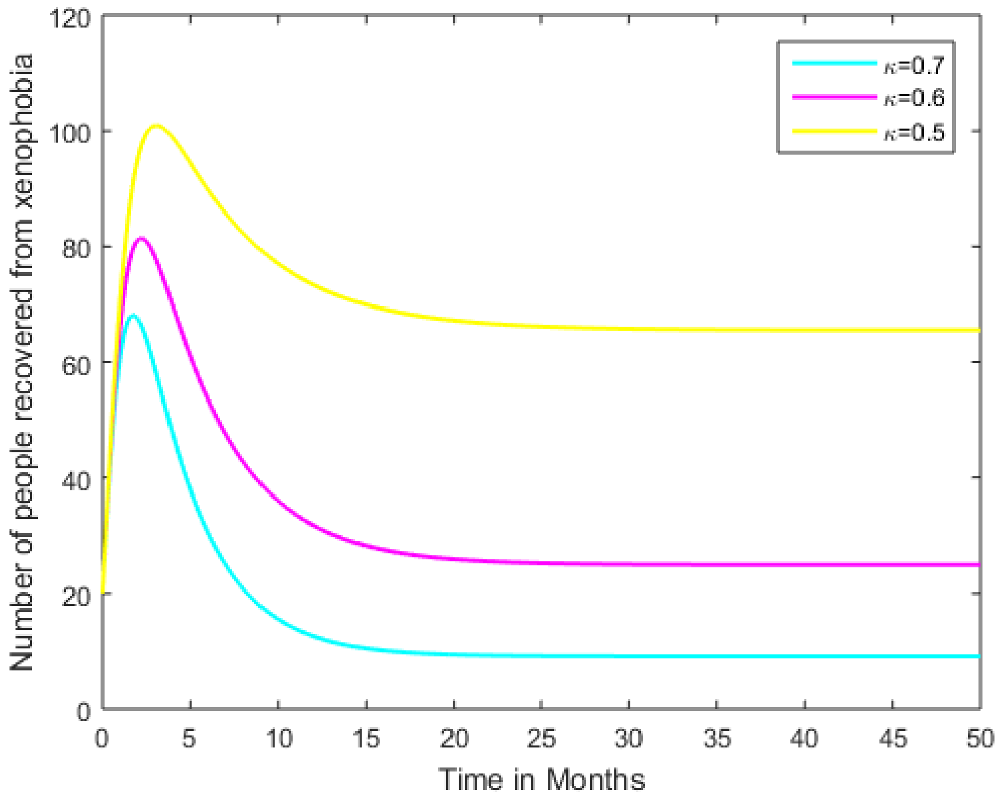


**Figure 9:** Dual intervention effect on individuals recovered from xenophobia

The plot in Figure 7 compares the evolution of xenophobic exposure under two scenarios: no intervention (w1=0, w2=0) and combined socio-economic and legal interventions (w1=w2=0.7). Without interventions, exposure levels remain persistently high throughout the 50-month period, demonstrating how unaddressed systemic issues perpetuate xenophobic vulnerability. In contrast, the dual-intervention scenario shows a significant decline, with exposure rates dropping steadily after an initial 10-month implementation period. By month 50, the intervention scenario maintains a stable low exposure level, suggesting these combined strategies create sustainable protection against xenophobic influences. The diverging trajectories emphasize that comprehensive approaches addressing both root causes and legal protections are far more effective than passive approaches in reducing community exposure to xenophobia.

The plot in Figure 8 compares xenophobia prevalence under two scenarios: no intervention (w1=0, w2=0) and combined socio-economic and legal interventions (w1=w2=0.7). Without interventions, xenophobia persists at high levels throughout the 50-month period, demonstrating the self-sustaining nature of prejudicial attitudes when systemic issues remain unaddressed. The intervention curve's sustained low plateau suggests these measures not only reduce but stabilize xenophobia rates. Compared to single-factor interventions, the dual approach proves substantially more effective, highlighting how comprehensive strategies that simultaneously address root causes and consequences outperform partial solutions. To investigate the impact of the parameter $\kappa$ on the dynamics of the fractional-order model presented in Equation (4.5), we performed multiple numerical simulations by varying the value of this parameter to observe its effects on the dynamics of public perception on xenophobic attitudes. Our goal was to examine the influence of memory (represented by the order of derivatives, $\kappa$) on the number of individuals exposed to xenophobic attitudes, affected by xenophobic attitudes, and recovered from being affected by xenophobic attitudes.

Research Paper
Open Access
ISSN: 3108-4052

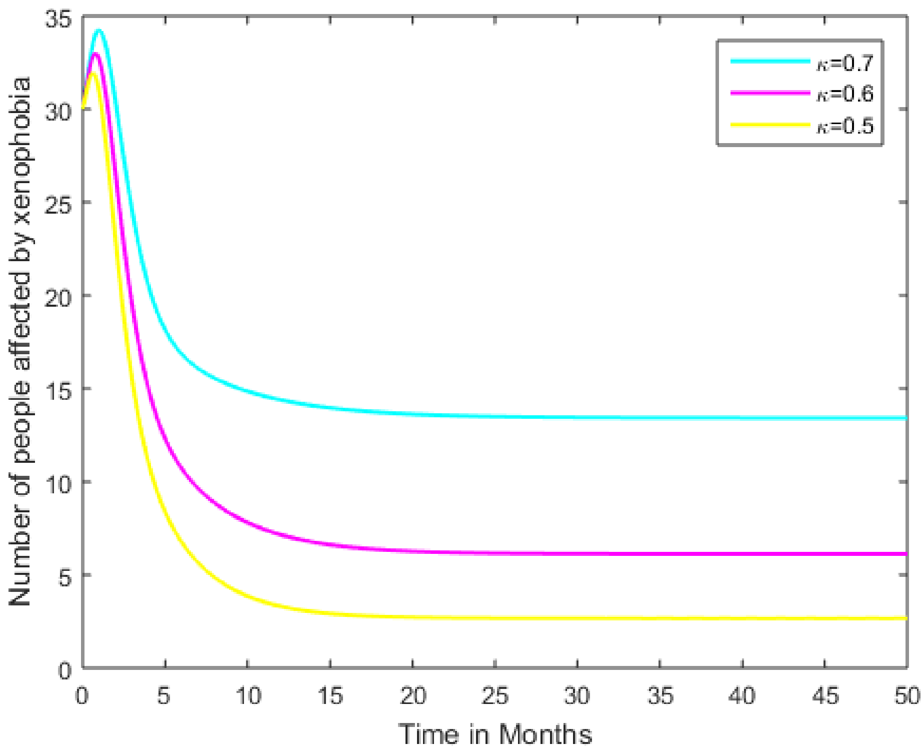


**Figure 10:** Dual intervention effect on individuals affected by xenophobia

The fractional order significantly influences the dynamics of individuals affected by and recovered from xenophobia, as illustrated in Figures 9 and 10. As the fractional order decreases, the recovery rate increases, leading to a higher number of individuals overcoming xenophobia, while the population of affected individuals declines. Figure 10 reveals that as the fractional order decreases, the affected population declines more rapidly, implying either enhanced recovery processes or a weakening of xenophobic tendencies over time. On the other hand, Figure 9 illustrates that a lower fractional order leads to an increase in the recovered population, suggesting that individuals' prior experiences and historical influences play a key role in their recovery from xenophobia. This highlights how memory effects and past interactions contribute to behavioral changes, with a smaller fractional order accelerating the shift from affected to recovered states. This inverse relationship highlights the critical role of the fractional order in shaping xenophobia-related outcomes—lower values promote recovery and reduce the prevalence of affected individuals, emphasizing the importance of adaptive strategies in mitigating xenophobic behaviors. The plots further validate that fractional calculus provides a nuanced understanding of these dynamics, capturing memory-dependent and non-local effects in behavioral transitions.

# 6 Conclusion

In this study, we conducted research on the evolution of xenophobic attitudes in the community. We developed a mathematical model, initially of an integer order and later reformulated it as a model of fractional order. The inclusion of fractional order equations allows for a more flexible and accurate representation of complex adaptive systems. We recalled essential concepts of fractional calculus that are integral to this research study.

Research Paper
Open Access
ISSN: 3108-4052

We analyzed the model by examining various aspects such as the non-negativity and boundedness of solutions, the existence and stability of equilibrium points using Routh-Hurwitz and Matignon's stability criteria. We also determined the xenophobic effective reproduction number using next generation matrix approach, and formulated an optimal control problem for the fractional order system.

Furthermore, our simulations using MATLAB programming showed that implementing both protection and treatment strategies proved to be the most effective approach in addressing the evolution of xenophobic attitudes and that examined the effect of memory on the xenophobia dynamics. We believe that this fractional order model provides a better understanding of the phenomenon compared to classical models, although further analysis and modifications by other researchers are encouraged.

It is important to note that the analysis using fractional derivatives is more complex than traditional deterministic modeling. This study represents the first exploration of the evolution of xenophobic attitudes in a fractional-order modeling approach.

Ultimately, the authors likely recommend stakeholders to prioritize measures to combat xenophobia effectively. The key strategies involve protection and treatment designed to counteract xenophobic attitudes at the right time and intensity. For instance, xenophobic rhetoric tends to surge during election cycles, as politicians or media outlets may exploit fears around cultural differences. With this insight, policymakers could strategically ramp up public awareness campaigns during these high-risk periods. This study can serve as a foundation for future research, which could incorporate additional concepts such as a stochastic approach, age structure, media effects, and real-world data fitting.

# Declarations

**Acknowledgements:** We extend our profound gratitude to the anonymous peer reviewers whose insightful critiques and scholarly suggestions have substantially elevated the quality of this manuscript. Their rigorous engagement with our work has proven invaluable in refining both its content and presentation.

**Author's Contributions:** Conceptualization, Formal Analysis, Investigation, Methodology, Re- sources, Software, Supervision, Visualization, Writing–original draft, Writing–review and editing., G.K.; Conceptualization, Formal Analysis, Investigation, Method- ology, Resources, Software, Supervision, Visualization, Writing–original draft, Writing–review and editing, Sh.W.; Conceptualization, Formal Analysis, Investigation, Methodology, Re- sources, Software, Supervision, Visualization, Writing original draft, Writing review and editing, D.K. All authors have read and agreed to the published version of the manuscript.

**Conflict of Interest Disclosure:** The authors declare no conflict of interest exists.



**Supporting/Supporting Organizations:** This research received no external funding.

**Ethical Approval and Participant Consent:** This article does not contain any studies with human or animal subjects. It is declared that during the preparation process of this study, scientific and ethical principles were followed and all the studies benefited from are stated in the bibliography.

**Plagiarism Statement:**This article was scanned by the plagiarism program. No plagiarism detected.

**Availability of Data and Materials:** All the data used for the study are available from the corresponding author upon request.

Research Paper
Open Access
ISSN: 3108-4052

**Use of AI tools:** The author declares that he has not used Artificial Intelligence (AI) tools in the creation of this article.

**Funding:** This study did not receive any funding.

**ORCIDs**

Gizachew Kefelew Hailu https://orcid.org/0000-0001-6490-1164

Shewafera Wondimagegnhu Teklu https://orcid.org/0000-0002-9073-6075

Dejen Ketema Mamo https://orcid.org/0000-0002-7126-4711

## References

[1] M. Sanchez-Mazas and L. Licata, *Xenophobia: Social psychological aspects*, International encyclopedia of the social & behavioral sciences, **25**, (2015), 802–807. Available from: https://doi.org/10.1016/B978-0-08-097086-8.24031-2

[2] P. F. Omoluabi, *Psychological foundation of xenophobia*, IFE PsychologIA: An International Journal, **16**(2), (2008), 53–71. Available from: https://hdl.handle.net/10520/ejc-ifepsyc-v16-nsi1-a4

[3] E. Akowe et al.,*A novel malaria mathematical model: integrating vector and non-vector transmission pathways*, BMC Infectious Diseases, **25**(1), (2025), 322. Available from: https://doi.org/10.1186/s12879-025-10653-8

[4] D. T. Campbell and B. R. McCandless,*Ethnocentrism, xenophobia and personality*, Human Relations, **4**(2), (1951), 185–192. Available from: https://doi.org/10.1177/001872675100400204

[5] F. B. Nyamnjoh,*Insiders and outsiders: Citizenship and xenophobia in contemporary Southern Africa*, Zed Books, (2006). Available from: https://books.google.com.et/books?id=Ywsj_-0TkKUC

[6] T. Polzer and K. Takabvirwa, *'just crime'?: Violence, xenophobia and crime: discourse and practice*, South African Crime Quarterly, **33**, (2010), 3–10. Available from: https://doi.org/10.17159/2413-3108/2010/v0i33a878

[7] L. Karamanidou, *Violence against migrants in greece: beyond the golden dawn*, Ethnic and Racial Studies, **39**(11), (2016), 2002–2021. Available from: https://doi.org/10.1080/01419870.2015.1124124

[8] C. H. Vhumbunu et al., *Addressing xenophobia through peacebuilding processes in south africa: Successes, challenges and prospects*, African Journal of Peace and Conflict Studies, **11**(2), (2022), 7. Available from: https://hdl.handle.net/10520/ejc-aa_ubuntu1_v11_n2_1

[9] B. C. Agbata et al., *A mathematical model for transmission dynamics of covid-19 in nigeria and its post effects*, International Journal of Mathematical Analysis and Modelling, **7**(2), (2024). Available from: https://tnsmb.org/journal/index.php/ijmam/article/view/191

[10] B. C. Agbata et al., *Mathematical analysis of the transmission dynamics of malaria and tuberculosis co-infection with control strategies*, Engineering Reports, **7**(6), (2025), e70210. Available from: https://doi.org/10.1002/eng2.70210

[11] D. K. Mamo et al.,*Mathematical modeling and analysis of the co-dynamics of crime and drug abuse*, Scientific Reports, **14**(1), (2024), 26461. Available from: https://doi.org/10.1038/s41598-024-75034-8

[12] B. C. Agbata et al.,*Numerical solution of fractional order model of measles disease with double dose vaccination*, Dutse Journal of Pure and Applied Sciences, **10**(3b), (2024), 202–217. Available from: https://10.4314/dujopas.v10i3b.19

[13] B. C. Agbata et al., *Fractional–order analysis of malaria and tuberculosis co-dynamics: A laplace adomian decomposition approach*, Edelweiss Applied Science and Technology, **9**(4), (2025), 1675–1714. Available from: https://doi.org/10.55214/25768484.v9i4.6353

[14] B. A. Baba and B. Bilgehan, *Optimal control of a fractional order model for the covid–19 pandemic*, Chaos, Solitons & Fractals, **144**, (2021), 110678. Available from: https://doi.org/10.1016/j.chaos.2021.110678

[15] I. Podlubny,*Fractional differential equations: an introduction to fractional derivatives, fractional differential equations, to methods of their solution and some of their applications*, Mathematics in Science and Engineering, (1998). Available from: https://books.google.com.et/books?id=F1idlAEACAAJ

[16] I. Petr´a˘s, *Fractional-order nonlinear systems: modeling, analysis and simulation*, Springer Science & Business Media, (2011). Available from: https://doi.org/10.1007/978-3-642-18101-6

[17] C. Vargas-De-Le´on, *Volterra-type lyapunov functions for fractional-order epidemic systems*, Communications in Nonlinear Science and Numerical Simulation, **24**(1-3), (2015), 75-85. Available from: https://doi.org/10.1016/j.cnsns.2014.12.013

[18] L. C. de Barros et al., *The memory effect on fractional calculus: an application in the spread of covid-19*, Computational and Applied Mathematics, **40**, (2021), 1–21. Available from: https://doi.org/10.1007/s40314-021-01456-z

[19] R. B. Ogunrinde et al., *Dynamic model of covid-19 and citizens reaction using fractional derivative*, Alexandria Engineering Journal, **60**(2), (2021), 2001–2012. Available from: https://doi.org/10.1016/j.aej.2020.09.016

[20] P. Van den Driessche and J. Watmough,*Reproduction numbers and subthreshold endemic equilibria for compartmental models of disease transmission*, Mathematical biosciences, **180**(1-2), (2002), 29-48. Available from: https://doi.org/10.1016/S0025-5564(02)00108-6

[21] G. K. Hailu and S. W. Teklu, *Improving passengers' attitudes toward safety and unreliable train operations: analysis of a mathematical model of fractional order*, Frontiers in Applied Mathematics and Statistics, **10**, (2024), 1290494. Available from: https://doi.org/10.3389/fams.2024.1290494

[22] G. K. Hailu et al., *Insight into employees' perceptions on reform initiatives in public service organizations using fractional order derivatives with optimal control strategies*, Frontiers in Complex Systems, **3**, (2025), 1565736. Available from: https://doi.org/10.3389/fcpxs.2025.1565736

[23] S. W. Teklu, *Analysis of fractional order model on higher institution students' anxiety towards mathematics with optimal control theory*, Scientific Reports, **13**(1), (2023), 6867. Available from: https://doi.org/10.1038/s41598-023-33961-y

[24] G. T. Tilahun et al.,*Modelling and optimal control of typhoid fever disease with cost-effective strategies*, Computational and Mathematical Methods in Medicine, **2017**(1), (2017), 2324518. Available from: https://doi.org/10.1155/2017/2324518

[25] S. W. Teklu and B. B. Terefe, *Mathematical modeling analysis on the dynamics of university students animosity towards mathematics with optimal control theory*, Scientific Reports, **12**(1), (2022), 11578. Available from: https://doi.org/10.1038/s41598-022-15376-3

[26] G. K. Hailu and S. W. Teklu, *Investigation into hate speech dissemination dynamics in a community using fractional order modeling approach*, Research in Mathematics, **11**(1), (2024), 2386748. Available from: https://doi.org/10.1080/27684830.2024.2386748